Math 2534  Logic Worksheet Solutions

Put all work on another sheet of paper and be neat and complete in your work

Problem 1:
Given the following statements convert the following sentences into symbolic logic form:
Part A

P:  Ed goes camping
Q:  Mountain lions are near
R:  It is snowing

a) It is snowing and Ed goes camping.
   \[ R \land P \]

b) It is not true that mountain lions are near but Ed does not go camping.
   \[ \neg (Q \land \neg P) \equiv \neg Q \lor P \]

c) It is a clear day or Ed does not camp.
   \[ \neg R \lor \neg P \]

d) Either it is a clear day or mountain lions are near.
   \[ \neg R \oplus Q \]

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Part B   Convert the following symbolic logic into natural conversational English.

a) \((R \land Q) \lor \neg P\)
   It is snowing and the mountain lions are near, or Ed does not go camping

b) \(\neg (P \lor Q)\)
   Using DeMorgan's Law: Ed does not go camping and Mountains lion are not near.

c) \(Q \lor (P \land Q)\)
   Using the absorption law: Mountain lions are near.

d) \(R \oplus (P \land Q)\)
   Either it is snowing or Ed goes camping and mountain lions are near.
Problem 2: Using Truth Tables, verify the following: $\neg[(P \land Q) \lor \neg R] \equiv (\neg P \lor \neg Q) \land R$

Summarize your results with clear English sentences.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$R$</th>
<th>$\neg P$</th>
<th>$\neg Q$</th>
<th>$\neg R$</th>
<th>$P \land Q$</th>
<th>$\neg P \lor \neg Q$</th>
<th>$(P \land Q) \lor \neg R$</th>
<th>$\neg[(P \land Q) \lor \neg R]$</th>
<th>$(\neg P \lor \neg Q) \land R$</th>
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</thead>
<tbody>
<tr>
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According to my truth table my identity is tautology since the truth values for both the left side and the right side of my identity are identical as shown in the last two columns.

Problem 3:
Use DeMorgan’s Law to find the negation of the following sentences. Show how you use the symbolic logic and then restate the results in English.

a) Anna will hike or rock climb
Let $A$ be the statement, Anna will hike and $R$ be the statement, Anna will rock climb.
$\neg(A \lor R) \equiv \neg A \land \neg R$
Therefore the negation is: Anna will neither hike nor rock climb.

b) Laura will study and pass the test
Let $L$ be the statement, Laura will study and $T$ be the statement, Laura will pass the test.
$\neg(L \land T) \equiv \neg L \lor \neg T$
Therefore the negation is: Laura will not study or not pass the test.
Problem 4: Put the following into conditional propositional logic form.

a) I will take a nap if you do.
   A: I will take a nap
   B: You will take a nap
   \[ B \rightarrow A \]

b) You will feel better only if you take a nap.
   F: Feel Better
   N: Take a nap
   \[ F \rightarrow N \]

c) If you do not leave then I can not finish my work.
   L: You leave
   W: I finish my work
   \[ \neg L \rightarrow \neg W \]

d) The game is on or it is raining.
   G: Game is on
   R: It is raining
   \[ G \lor R \equiv \neg G \rightarrow R \]

e) You must exercise to build endurance.
   E: You exercise
   B: You build endurance
   \[ B \rightarrow E \]

Problem 5: The NAND operator is denoted by \( \mid \) and is defined by \( P \mid Q \equiv \neg(P \land Q) \)

a) Develop the truth table for this operator

\[
\begin{array}{ccc}
P & Q & P \land Q & \neg(P \land Q) \\
T & T & T & F \\
T & F & F & T \\
F & T & F & T \\
F & F & F & T \\
\end{array}
\]

b) Prove that \( P \mid P \equiv \neg P \) (do not use truth tables)

\[ \neg P \equiv \neg(P \land P) \equiv P \mid P \quad \text{By definition of NAND} \]

c) Develop the equivalent NAND representation for \( P \lor Q \)

\[
P \lor Q \equiv \neg \neg(P \lor Q) \equiv \neg(P \land \neg Q) \equiv \neg \neg P \mid \neg Q \equiv (P \mid P) \mid (Q \mid Q) \equiv
\]
Problem 6: Given the following statements:

P: Sara is a freshman at VA Tech
Q: UVA is the best School in Virginia
R: VA Tech is number one

Part A:
Suppose the statement P is true and Q is false. Express each of the following compound statements in symbolic logic notation and then determine its true value.

You could create truth tables but it is easier just to reason it out.

1) Sara is not a freshman at VA Tech but UVA is the best school in Virginia.
   \[ \neg P \land Q \equiv F \land F = F \]

2) Either Sara is a freshman at VA Tech or UVA is not the best school in Virginia.
   \[ P \oplus \neg Q \equiv T \oplus T = F \]

Part B:

If we assume that the statements P, Q and R have no assigned truth values but it is given that the implication \( R \rightarrow (P \lor Q) \) is false then (if possible) determine the truth values for each statement.

Solution: In order for the implication statement to be false the sufficient condition R must be true and the necessary condition \( (P \lor Q) \) be false. Since the necessary condition is false then both P and Q are false since it is an “or” statement.