Problem 1:  
Theorem: If a does not divide \( b + c \), then a does not divide b or does not divide c. (Hint: use contrapositive)

Problem 2:  
Theorem: For all natural numbers \( n^2 - n \) is even. (Hint: factor)

Problem 3: (use method of contradiction)  
Theorem: If \( x \) is a real number and \( x^3 + 4x = 0 \), then \( x = 0 \).

Problem 4: (use method of contradiction)  
Theorem: The quotient of a non zero rational number divided by an irrational number is always irrational.

Problem 5:  
Theorem: The product of any 3 consecutive integers is always divisible by 3.

Problem 6: If \( a \mod 5 = 2 \), then find the value of \( (8a) \mod 5 \).

Problem 7: Given the following puzzle where \( a, b \) and \( c \) are integers and \( a^2 + b \) is odd and \( a - c \) is even with \( c \) odd. Then determine if \( b + c \) is odd or even. Next formulate a theorem concerning your results and prove it.