Problems On Functions for Class Discussion

Problem 1:
If \( A = \{0,1,2,3,4\} \) \( B = \{1,3,4,6\} \) and

\[
\begin{align*}
G(f) &= \{(0,6), (1,3), (2,1), (3,3), (4,4)\} \\
G(g) &= \{(1.4), (3,2), (4.0), (6,3)\}
\end{align*}
\]

a) Find \( G(f \circ g) \)  
   b) Find \( G(f / g) \)
   c) Is \( f \) one to one? Why or why not?
   d) is \( f \) onto? Why or why not?
   e) Is \( f \circ g \) one to one? Onto? Why or why not?
   d) does \( g^{-1} \) exist? Why or why not?
   e) find \( g^{-1} \circ g \) and describe the domain and range

Problem 2:
A) Given that \( f(x) = \frac{x}{5x+1} \), state the domain.
B) Is \( f(x) \) one to one? Verify your answer.
C) Is \( f(x) \) onto? Verify your answer.
D) Does \( f^{-1} \) exist? Verify your answer. If it does not exist, modify the domain so that an inverse function exist.

Problem 3:
Theorem: If the A and B are finite sets such that the \( n(A) < n(B) \), then any function mapping A to B can not be onto.

Problem 4:
Theorem: If \( f(x) \) and \( g(x) \) are each onto functions, then the composition \( g \circ f \) is also onto.

Problem 5:
Theorem: If \( f(x) \) and \( g(x) \) are each one to one functions, then the composition \( g \circ f \) is also one to one.

Problem 6:
Theorem: If the A and B are finite sets such that the \( n(A) = n(B) \), then any function mapping A to B is one to one if and only if it is onto.

Problem 7:
If \( f: (\mathbb{Z} \mod 5) \to (\mathbb{Z} \mod 4) \) when \( f[x] = [3x + 2] \), determine if \( f \) is a bijection.

Problem 8:
Theorem: Any function defined on the integers is a bijection if and only if the inverse function exist.
Problem 9:
Explain the mistake in the following proof:
Theorem: If \( f(x) = 4x + 3 \) for all integers, Then \( f(x) \) is one to one.
Proof: Suppose any integer \( x \) is given. Then by definition of \( f \), there is only one possible value for \( f(x) \), namely \( 4x + 3 \). Hence \( f(x) \) is one to one.

Problem 10:
Define If \( A = \{a, b, c\} \), then define \( F : P(A) \rightarrow \mathbb{Z} \) as follows: For all subsets \( S \) in \( P(A) \), \( F(S) = n(S) \) (ie. the number of elements in \( S \))

a) Is \( F \) one to one? Prove or give a counter example
b) Is \( F \) onto? Prove or give a counter example

Problem 11:
Let \( A \) and \( B \) be finite sets and \( n(A) > n(B) \). If \( f \) maps \( A \) to \( B \), then \( f \) can not be one to one.