Problem 1: Use Algebra of Logic to Prove the following:

a) Theorem 1: \((\neg p \lor q) \land (p \lor \neg r) \land (\neg q \lor \neg r) \equiv \neg p \land \neg r\)

Proof:
\[
(\sim p \lor q) \land (p \lor \sim r) \land (\sim q \lor \sim r) \equiv \text{given}
\]
\[
[(\sim p \lor q) \land (\sim p \lor q)] \land (p \lor \sim r) \equiv \text{by Commutative and Associative laws}
\]
\[
[\sim p \lor (q \land \sim q)] \land (p \lor \sim r) \equiv \text{by Distributive Law}
\]
\[
[\sim p \lor (F)] \land (p \lor \sim r) \equiv \text{by Inverse Law}
\]
\[
[\sim p] \land (p \lor \sim r) \equiv \text{by Identity Law}
\]
\[
(\sim p \land p) \lor (\sim p \land \sim r) \equiv \text{by Distributive Law}
\]
\[
(F) \lor (\sim p \land \sim r) \equiv \text{by Inverse Law}
\]
\[
(\sim p \land \sim r) \equiv \text{by Identity Law}
\]
\[
\therefore (\sim p \lor q) \land (p \lor \sim r) \land (\sim q \lor \sim r) \equiv (\sim p \land \sim r)
\]

b) Theorem 2: \([\sim(\sim p \rightarrow q) \lor (\sim p \land \sim q)] \rightarrow (p \land q) \equiv p\)

Proof:
\[
[\sim(\sim p \rightarrow q) \lor (\sim p \land \sim q)] \rightarrow (p \land q) \equiv \text{given}
\]
\[
\sim [\sim (\sim p \lor q) \lor (\sim p \land \sim q)] \lor (p \land q) \equiv \text{Implication Law}
\]
\[
\sim [(\sim p \lor q) \lor (\sim p \land \sim q)] \lor (p \land q) \equiv \text{Double Negative Law}
\]
\[
[(\sim p \lor q) \lor (\sim p \land \sim q)] \lor (p \land q) \equiv \text{DeMorgan's Law}
\]
\[
[(p \lor \sim q) \lor (\sim p \land \sim q)] \lor (p \land q) \equiv \text{Double Negative Law}
\]
\[
[(p \lor \sim q) \lor (\sim p \land \sim q)] \lor (p \land q) \equiv \text{DeMorgan's Law}
\]
\[
[(p \lor \sim q) \lor (p \land q)] \lor (p \land q) \equiv \text{Double Negative Law}
\]
\[
[(p \lor \sim q) \lor (p \land q)] \lor (p \land q) \equiv \text{Distributive Law}
\]
\[
[(p \lor (F)) \lor (p \land q)] \lor (p \land q) \equiv \text{Inverse Law}
\]
\[
p \lor (p \land q) \equiv \text{Identity Law}
\]
\[
p \lor (p \land q) \equiv \text{Absorption Law}
\]
\[
\therefore [\sim(\sim p \rightarrow q) \lor (\sim p \land \sim q)] \rightarrow (p \land q) \equiv p
\]
Problem 2: Put the following into implication form. Define all your variables.

a) You will be in the band only if you practice everyday.
   Let B be the statement “You will be in the band
   Let P be the statement “you practice”
   \( B \rightarrow P \)

b) Do not go to the party or you will not pass the test.
   Let P be the statement “you go to the party”
   Let T be the statement “you pass the test”
   \( \neg P \lor \neg T \equiv P \rightarrow \neg T \)

Problem 3: Are any the following statements equivalent? Put into symbolic logic and justify your reasoning. Define your variables.

a) I will study if you study.
   Let I be the statement “I will study”.
   Let Y be the statement “you will study”.
   \( Y \rightarrow I \)

b) Only if you study will I study.
   \( I \rightarrow Y \)

c) You do not study or I do not study.
   \( \neg Y \lor \neg I \equiv \neg Y \rightarrow \neg I \)

d) You study when I study.
   \( I \rightarrow Y \)

Part b) and d) are equivalent since the sufficient and necessary conditions are the same.

Problem 4: Given the statement: The door is open only if the professor is having office hours.

a) Put this statement into symbolic logic notation
   Let O be the statement “the door is open”.
   Let P be the statement “the Professor is having office hours”.
   \( O \rightarrow P \)

b) Negate the symbolic logic in part a and then put into an English sentence.
   \( \neg (O \rightarrow P) \equiv (\neg O \lor P) \equiv O \land \neg P \)
   The door is open and the professor is not holding office hours.
Problem 5: Determine if the following arguments are valid and justify your conclusion.

a) If you miss this class, you will not do well.
   You did not do well in this class.
   Therefore you did miss class.

Let $C$ be the statement “you miss the class”.
Let $W$ be the statement “you do well”.

$C \rightarrow \neg W \quad \neg W \quad \therefore C$  

The argument is not valid. The necessary condition does not guarantee the sufficient condition.  
This is a converse error.

b) If it snows, school will be canceled.
   School was not canceled
   Therefore it did not snow.

Let $S$ be the statement “It snows”.
Let $C$ be the statement “School is canceled”.

$S \rightarrow C \quad \neg C \quad \therefore \neg S$  

The argument is valid by the contrapositive which is equivalent to the original implication.

c) You will take the train if I do.
   I do not take the train.
   Therefore you do not take the train.

Let $Y$ be the statement “you take the train”.
Let $I$ be the statement “I take the train”.

$I \rightarrow Y \quad \neg I \quad \therefore \neg Y$  

The argument is not valid. This is the inverse form which not equivalent to the original implication.  This is the inverse error.
Problem 6:
The logic puzzle below is adapted from Lewis Carroll the author of Alice in Wonderland.

Using symbolic logic, put the following sentences in implication form and then supply the conclusion that will make this argument valid. Show all work and justify your reasoning in paragraph form by referring to the sufficient and necessary conditions. All statements are consider to be true.

If he goes to the party, he does not fail to brush his hair. To look debonair it is necessary to be tidy. If he drinks then he has no self-control. If he does not look debonair, he did not brush his hair. He wears white gloves only if he goes to the party which is a black tie affair. Having no self-control is sufficient to make one untidy. Therefore ……………….

Define your variables as follows:
P: he goes to the party
B: he brush his hair
D: he looks debonair
T: he is tidy
R: he drinks
S: he has self-control
W: he wears white gloves

Solutions:
Given the statements above:
\[ P \rightarrow B \]
\[ D \rightarrow T \]
\[ R \rightarrow \neg S \equiv S \rightarrow \neg R \quad \text{by contrapositive} \]
\[ \neg D \rightarrow \neg B \equiv B \rightarrow D \quad \text{by contrapositive} \]
\[ W \rightarrow P \]
\[ \neg S \rightarrow \neg T \equiv T \rightarrow S \quad \text{by contrapositive} \]

Using the Transitive argument form we have the following:
\[ W \rightarrow P \]
\[ P \rightarrow B \]
\[ \therefore W \rightarrow B \]
\[ W \rightarrow B \]
\[ B \rightarrow D \]
\[ \therefore W \rightarrow D \]
\[ W \rightarrow D \]
\[ D \rightarrow T \]
\[ \therefore W \rightarrow T \]

\[ W \rightarrow T \]
\[ T \rightarrow S \]
\[ \therefore W \rightarrow S \]

\[ W \rightarrow S \]
\[ S \rightarrow \neg R \]
\[ \therefore W \rightarrow \neg R \]

If he is wearing white gloves, then he is not drinking.