Math 2534   Equivalence Relations and the Pigeon Hole Principle

1) For each of the following relations defined on the set A = {1,2,3,4,5}, determine if R is reflexive, symmetric and/or transitive. Draw the directed graphs for each relation. If R is an equivalence relation then represent A as a partition.

\[ R_1 = \{(1,1),(2,2),(2,3),(3,2),(3,3),(5,4),(4,3),(3,4),(4,4),(5,3),(4,2),(5,2)\} \]
\[ R_2 = \{(1,1),(1,4),(4,1),(2,2),(4,3),(3,4),(3,3),(5,5),(4,4)\} \]
\[ R_3 = \{(2,4),(3,1),(3,5),(4,2),(2,2),(4,4),(5,1),(5,5)\} \]
\[ R_4 = \{(1,1),(2,2),(1,3),(1,5),(5,1),(5,5)\} \]

2) Prove the following:

A) Theorem: Given positive integers \(a, b\), then \(a \equiv b \mod 3\) is an equivalence relation on the integers. Define a relation R by \( mRn \) if and only if \(3 | m - n\).

B) Theorem: If \(a, b\) are integers, then R is an equivalence relation when \(aRb\) if and only if \(a + b\) is even.

C) 1) Theorem: If S and R are each transitive relations then \(S \cap R\) is also transitive.

2) Give a counter example to show that \(S \cup R\) is not transitive.

3) If an equivalence relation is defined by the following set partition on A, then express R as a set of ordered pairs. Also draw a directed graph that illustrates R.
\[ A = \{2,4,5\} \cup \{3,6\} \cup \{1\} \]

4) How many students have to be in the same class to guaranteed that 13 students in that class have the same last initial?