Math 2534  Homework 10 on Functions, Spring 2012

Problem 1: If the functions f and g are both surjections then the composition \((g \circ f)\) is also a surjection. For the proof assume \(f : A \to B\) and \(g : B \to C\).

Problem 2: Prove that if \(F\) is onto, then \(F^{-1}\) is defined for each element in its domain.

Problem 3: If \(f\) maps finite sets \(A\) to \(B\) and \(n(A) > n(B)\) prove that \(f\) cannot be an injection.

Problem 4: Define If \(A = \{a, b, c, d\}\), then define \(F : P(A) \to \{0, 1, 2, 3, 4, 5, 6\}\) as follows: For all subsets \(S\) in \(P(A)\), \(F(S) = n(S)\) (i.e. the number of elements in \(S\)).
   a) Is \(F\) one to one? Justify your conclusion
   b) Is \(F\) onto? Justify your conclusion

Problem 5: If \(f : (\text{zmod}5) \to (\text{zmod} 5)\) when \(f[x] = [2x + 1]\), determine if \(f\) is a bijection.

Problem 6: 4) Determine if the following statements are true or false.
   a) “The function \(f\) is onto” \(\iff\) “Every element in the co-domain is the image of some element in the domain”.
   b) “The function \(f\) is onto” \(\iff\) “Every element in the domain has an image in the co-domain”.
   c) “The function \(f\) is onto” \(\iff\) “\(\forall y \in Y, \exists x \in X \ni f(x) = y\)”
   d) “The function \(f\) is onto” \(\iff\) “\(\forall x \in X, \exists y \in Y \ni f(x) = y\)”
   e) “The function \(f\) is onto” \(\iff\) “the range and co-domain is the same set”

Problem 7: Given the function \(f(x) = \frac{x}{x-3}\),
   a) Using the precise definition of injection, determine if \(f(x)\) is one to one.
   b) Using the precise definition of surjection determine if \(f(x)\) is onto.
   c) Does the inverse of \(f(x)\) exist? If not, then make modifications necessary to produce an inverse function.