Solution to EC:

Theorem: For all natural numbers $1 + a + a^2 + a^3 + ... + a^{n-1} = \frac{a^n - 1}{a - 1}$

Proof: The hypothesis is true for at least one value of $n$. Consider $n = 1$ which give us that

$$1 = \frac{a - 1}{a - 1}.$$ 

We will also consider $n = 2$ in order to see the pattern.

Notice that

$$1 + a = \frac{a^2 - 1}{a - 1} = \frac{(a - 1)(a + 1)}{a - 1} = a + 1.$$ 

Now assume that the hypothesis is true from $n = 2$ up to some arbitrary value $k$ so that

$$1 + a + a^2 + a^3 + ... + a^{k-1} = \frac{a^k - 1}{a - 1}$$

and prove true for $k + 1$ by showing that

$$1 + a + a^2 + a^3 + ... + a^k = \frac{a^{k+1} - 1}{a - 1}.$$ 

For the body of the proof, consider the $k + 1$ term, $1 + a + a^2 + a^3 + ... + a^k$.

By the inductive hypothesis we have that

$$1 + a + a^2 + a^3 + ... + a^{k-1} + a^k =$$

$$\frac{a^k - 1}{a - 1} + a^k$$

using algebra we have that

$$\frac{a^k - 1}{a - 1} + a^k = \frac{(a^k - 1) + (a^k)(a - 1)}{a - 1} = \frac{(a^k - 1) + (a^{k+1} - a^k)}{a - 1} = \frac{(a^{k+1} - 1)}{a - 1}$$

so

$$1 + a + a^2 + a^3 + ... + a^k = \frac{a^{k+1} - 1}{a - 1}$$

Since we assumed true up to $k$ and proved true for $k + 1$, the hypothesis is true for all natural numbers.