<u>Math 2114 solutions Homework 9 Spring 2018</u>

Follow homework requirements to get full credit.

Problem 1: Given the information matrix $\begin{bmatrix} \textit{from} & & \textit{to} \\ \textit{city} & \textit{suburbs} & \\ .95 & .03 & \textit{city} \\ .05 & .97 & \textit{suburbs} \end{bmatrix}$,

let M = $\begin{bmatrix} .95 & .03 \\ .05 & .97 \end{bmatrix}$ be the transition matrix that illustrates the movement between populations in the city and suburbs. Total population is 600,000 in the city and 400,000 in the suburbs.

Now choose one arbitrary person in the city and determine the likely hood that he will move to the suburbs in one year and then determine the likelihood he will move in two years. Let the initial vector $x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

Solution:

First year $\begin{bmatrix} .95 & .03 \\ .05 & .97 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} .95 \\ .05 \end{bmatrix}$

Second year $\begin{bmatrix} .95 & .03 \\ .05 & .97 \end{bmatrix}\begin{bmatrix} .95 \\ .05 \end{bmatrix} = \begin{bmatrix} .904 \\ .096 \end{bmatrix}$

The likelihood of this person moving to the suburbs after one year is .05.
The likelihood of this person moving to the suburbs after two years is .096.
The likelihood is higher after two years.

Problem 2: Determine if $q = \begin{bmatrix} .3 \\ .7 \end{bmatrix}$ is a steady-state vector for $P\begin{bmatrix} .6 & .2 \\ .4 & .8 \end{bmatrix}$.

Solution:

$$\begin{bmatrix} .6 & .2 \\ .4 & .8 \end{bmatrix}\begin{bmatrix} .3 \\ .7 \end{bmatrix} = \begin{bmatrix} .32 \\ .68 \end{bmatrix}$$

since $Aq \neq q$, q is not the steady state vector and more calculations are needed.

Problem 3: Give the matrix $A = \begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}$

1) Find the eigenvalues and eigenvectors

   Set $\det\begin{bmatrix} 1-\lambda & 5 \\ -2 & 3-\lambda \end{bmatrix} = 0$, To get that $\lambda = 2 \pm 3i$

2) Using the eigenvector associated with $\lambda =$ a – bi for A, find the matrix P and P^{-1} so that the rotation matrix $C = P^{-1}AP$

   To find the eigenvectors solve the system

   $$\begin{bmatrix} -1+3i & 5 & 0 \\ -2 & 1+3i & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -(1/2)-(3/2)i & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

   $$eigenvector: \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} (1/2)+(3/2)i \\ 1 \end{bmatrix}$$

   $$P = \begin{bmatrix} 1/2 & 3/2 \\ 1 & 0 \end{bmatrix} \quad \mathrm{P}^{-1} = 2/3 \begin{bmatrix} 0 & -3/2 \\ -1 & 1/2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -2/3 & 1/3 \end{bmatrix}$$

   $$C = P^{-1}AP = \begin{bmatrix} 2 & -3 \\ 3 & 2 \end{bmatrix}$$

3) Using C from part 2), find the angle of rotation for the linear transformation T(x) = Ax. Remember that C and A are similar matrices and have the same eigenvalues.

   Since $\lambda = 2-3i$, $\theta = \mathrm{Tan}^{-1}(3/2)$ = .982 rad $\approx$ 56.30 degrees