<u>Math 2114 Solution Homework 2 on Poole 2.1, 2.2 and Larson 2.1</u>

Show all work and follow the homework requirements. Staple all multiple sheets.

Problem 1: Using the formula

$p = \text{Proj}_u v = \dfrac{v \bullet u}{u \bullet u} u,$ where vector p is the projection of v onto u and $u = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ and $v = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$

Solution:

$$u \bullet v = u^T v = \begin{bmatrix} 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix} = -4$$

$$u \bullet u = u^T u = \begin{bmatrix} 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = 6$$

$$\text{Proj}_u v = \frac{v \bullet u}{u \bullet u} u = \frac{-4}{6} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \frac{-2}{3} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2/3 \\ 4/3 \\ -2/3 \end{bmatrix}$$

Problem 2: Determine if the following augmented matrices are in Reduced Row Echelon Form. If not then continue the reduction to final results.

A) $$\begin{bmatrix} 1 & 0 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 2 \end{bmatrix} \rightarrow reduce \rightarrow \begin{bmatrix} 1 & 0 & 0 & 4 & 0 & 6 & -2 \\ 0 & 1 & 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 2 \end{bmatrix}$$

B) $$\begin{bmatrix} 0 & 1 & 2 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 2 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} R_4 \leftrightarrow R_2 \;.$$

Problem 3: Put this system in **Reduced Row Echelon Form** using the Gauss-Jordan method and solve: Give the solutions in vector form and indicate any free variables. State the Rank.

$$\begin{cases} x_1 - x_2 + 2x_3 - x_4 = 0 \\ 2x_1 + x_2 - 2x_3 - 2x_4 = 0 \\ -x_1 + 2x_2 - 4x_3 = 0 \\ 3x_1 - 3x_4 = 0 \end{cases}$$

$$\begin{bmatrix} 1 & -1 & 2 & -1 & 0 \\ 2 & 1 & -2 & -2 & 0 \\ -1 & 2 & -4 & 0 & 0 \\ 3 & 0 & 0 & -3 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 2 & -1 & 0 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & -1 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{bmatrix} \begin{matrix} \\ -2R_1 + R_2 \\ R_1 + R_3 \\ -3R_1 + R_4 \end{matrix} \to \begin{bmatrix} 1 & -1 & 2 & -1 & 0 \\ 0 & 1 & -2 & -1 & 0 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{bmatrix} R_3 \leftrightarrow R_2$$

$$\to \begin{bmatrix} 1 & 0 & 0 & -2 & 0 \\ 0 & 1 & -2 & -1 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 & 0 \end{bmatrix} \begin{matrix} R_2 + R_1 \\ \\ -3R_2 + R_3 \\ -3R_2 + R_4 \end{matrix} \to \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} x_1 = 0 \\ x_2 = 2x_3 \\ x_4 = 0 \end{matrix} \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 2x_3 \\ x_3 \\ 0 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

Rank is 3 and one free variable. Solution space is dimension 1 and is a straight line that passes through the origin.

Problem 4: Put this system in **Reduced Row Echelon Form** using the Gauss-Jordan method and solve. Give the solutions in vector form and indicate any free variables. State the Rank.

$$\begin{cases} x_1 + 6x_2 + 2x_3 - 5x_4 - 2x_5 = -4 \\ 2x_3 - 8x_4 - x_5 = 3 \\ 2x_3 - 8x_4 = 10 \end{cases}$$

$$\begin{bmatrix} 1 & 6 & 2 & -5 & -2 & -4 \\ 0 & 0 & 2 & -8 & -1 & 3 \\ 0 & 0 & 2 & -8 & 0 & 10 \end{bmatrix} \to RREF \to \begin{bmatrix} 1 & 6 & 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & -4 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -6x_2 - 3x_4 \\ x_2 \\ 4x_4 + 5 \\ x_4 \\ 7 \end{bmatrix} = x_2 \begin{bmatrix} -6 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -3 \\ 0 \\ 4 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \\ 0 \\ 7 \end{bmatrix}$$

Rank is 3 with 2 free variables

Problem 5: Given the system below,

$$\begin{cases} x + ny = 2 \\ -2x + 2y = p \end{cases}$$

Set up the augmented matrix for this system and do enough reduction to help answer the questions below.

$$\begin{bmatrix} 1 & n & 2 \\ -2 & 2 & p \end{bmatrix} \rightarrow \begin{bmatrix} 1 & n & 2 \\ 0 & 2n+2 & p+4 \end{bmatrix}$$

p

Choose n and p so that :

1) the system has no solution. $n = -1$ and $p \neq -4$

2) the system has a unique solution. $n \neq -1$, p can be real value

3) the system has infinite solution. $n = -1 \; and \; p = -4$

Problem 6: Determine if the vectors $v_1 = \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$ are Linearly Independent setting up the appropriate system according to the definition of Linearly Independent and reducing the augmented matrix.

We need to show that when $C_1 \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix} + C_2 \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix} + C_3 \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ the coefficients must be zero only.

Set up the augment matrix and solve for the coefficients

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ -3 & 0 & 3 & 0 \\ 0 & 3 & 1 & 0 \end{bmatrix} \text{reduced row echelon} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$so \;\; C_1 = C_2 = C_3 = 0$

So these vectors are linearly independent.