<u>Math 2114 Solutions Homework 10 Spring 2018 (sec 5.1 – 5.3)</u>

Show all work and staple multiple sheets.

Problem 1: Determine if the vectors below form an orthogonal set.

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, v_4 = \begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \end{bmatrix}$$

Give the dimension of the subspace W that can be spanned by these vectors as well as the larger space that contains W.

Solution:

$v_2 \bullet v_4 = 3 \neq 0$ so this set of vectors is not orthogonal.

Set up matrix A with column vectors above

$$A = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 1 & 1 & 1 \\ 1 & 2 & 0 & 1 \end{bmatrix} \to RREF \to \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 3 & 0 & 1 \end{bmatrix}$$

So they are linearly independent and span all of R^4.

Problem 2: Let a subspace W be defined below:

$$W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{ so that } x_1 + x_2 - x_3 = 0 \right\}$$

a) Find the basis for subspace W.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_2 + x_3 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\beta = \left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\} \text{ and W has dimension 2 in } R^3$$

b) Find the basis for the orthogonal complement $W^{\perp}$ of W.

(Hint: $W^{\perp} = \left(Col\left(A\right)\right)^{\perp} = Null(A^T)$ where the columns of A are the basis of W)

$$Let\ \ A = \begin{bmatrix} -1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } A^T = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

No Solve the system $A^T X = 0$

$$\begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \rightarrow RREF \rightarrow \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \quad \begin{matrix} x_1 - x_3 = 0 \\ x_2 + x_3 = 0 \end{matrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ -x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \quad \text{so } \beta = \left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right\} \text{ and } W^{\perp} \text{ has dimension 1 in } R^3$$

Notice that subspaces W union $W^{\perp}$ will equal the entire vector space R^3.

Problem 3: Let the vectors given below be a basis for a subspace W in R^4.

$$v_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} 3 \\ -1 \\ 0 \\ 4 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Use this basis the Gram-Schmidt Process to obtain an orthogonal basis for the subspace W.

<u>Solution:</u> (I have not had time to check my arithmetic. The process is correct. Let me know if you find errors)

$$\text{let } u_1 = v_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \\ 2 \end{bmatrix} \qquad \mathrm{W}_1 = span\{u_1\}$$

$$u_2 = v_2 - proj_{w_1} v_2 = \begin{bmatrix} 3 \\ -1 \\ 0 \\ 4 \end{bmatrix} - \left(\frac{v_2 \bullet u_1}{u_1 \bullet u_1} \right) u_1 = \begin{bmatrix} 3 \\ -1 \\ 0 \\ 4 \end{bmatrix} - \frac{15}{10} \begin{bmatrix} 2 \\ -1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1/2 \\ -3/2 \\ 1 \end{bmatrix} \qquad \mathrm{W}_2 = span\{u_1, u_2\}$$

$$Choose \ \mathrm{u}_2 = \begin{bmatrix} 0 \\ 1 \\ -3 \\ 2 \end{bmatrix}$$

$$u_3 = v_3 - proj_{w_2} v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \left(\frac{v_3 \bullet u_1}{u_1 \bullet u_1} \right) u_1 - \left(\frac{v_3 \bullet u_2}{u_2 \bullet u_2} \right) u_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \frac{4}{10} \begin{bmatrix} 2 \\ -1 \\ 1 \\ 2 \end{bmatrix} - \frac{0}{14} \begin{bmatrix} 0 \\ 1 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 1/5 \\ 7/5 \\ 3/5 \\ 1/5 \end{bmatrix}$$

$$\text{Choose } u_3 = \begin{bmatrix} 1 \\ 7 \\ 3 \\ 1 \end{bmatrix} \text{ for the orthogonal basis } \beta = \left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 7 \\ 3 \\ 1 \end{bmatrix} \right\}$$