Math 2114 Test 2 Fall 2017 Name___________________________

No Electronic devices for any kind. Show all work for full credit. **Pledge work when done.**

Problem 1: Let $T(v) = Av = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 7 & 2 & -1 & 3 \end{bmatrix} v$ be a linear transformation.

Answer the following questions:

a) Find the domain and co-domain so that $R^n \to R^m$

b) Determine if columns of the standard matrix are linearly independent.
(Justify your conclusion)

c) Determine if T is an **onto** mapping.
(Justify your conclusion)

d) Determine if T is a **one to one** mapping
(Justify your conclusion)

e) Evaluate Is the vector $\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$ in the range of T?

Problem 2: Using cofactor expansion only, find the value k so that the det(A) = 7.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 3 \\ 2 & 1 & k \end{bmatrix}$$

Problem 3: Find the determinate below when given that the $\det(\text{A}) = 4$.

1) $\det(3\,\text{A}^{-1})$
2) $\det(2A^T)$

Problem 4: Let T(v) = Av and $A_{5\times5}$ and the $\det A \neq 0$. Describe the following subspaces and justify your assertions.

1) Col A
2) Kernel

Problem 5:

Below is a system of first order linear differential equations.

$$Y' = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix} Y$$

Given the eigenvalues are $\lambda = 1, \ \lambda = -1, \ \lambda = 4$ and the eigenvectors for

$$\lambda_1 = 1, \ E_1 = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}, \qquad \lambda_2 = -1, \ E_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

a) Find the eigenvector E_3 for $\lambda_3 = 4$.

b) Give the dimension of the eigenspace spanned by $E_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$ and give an example of another eigenvector **u** in this space. Use your new vector **u** to verify that $Au = \lambda u$.

Problem 6: Given the matrix $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ and using matrix $P = \begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix}$, verify that A is diagonalizable.

1) Using the information above, give the eigenvalues and eigenvectors associated with A.
2) Using iteration, find the product $A^3 v$, when v = $\begin{bmatrix} 3 \\ -1 \end{bmatrix} = 2E_1 + E_2$ and $A^k v = \lambda^k v$.