Math 2114 Solution Test 1 Spring 2018

Problem 1: If $A=\begin{bmatrix}1 & 1 & -1\\ -1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$ and $B=\begin{bmatrix}3\\ 1\\ -2\end{bmatrix}$, solve the system $AX=B$ using A^{-1}.

Solution (14pts)

$$\begin{bmatrix}1 & 1 & -1 & 1 & 0 & 0\\ -1 & 0 & 1 & 0 & 1 & 0\\ 1 & 1 & 0 & 0 & 0 & 1\end{bmatrix} \to \text{show work} \to \begin{bmatrix}1 & 0 & 0 & -1 & -1 & 1\\ 0 & 1 & 0 & 1 & 1 & 0\\ 0 & 0 & 1 & -1 & 0 & 1\end{bmatrix} =$$

so we have that $A^{-1}=\begin{bmatrix}-1 & -1 & 1\\ 1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}$

$$X=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=A^{-1}B=\begin{bmatrix}-1 & -1 & 1\\ 1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}\begin{bmatrix}3\\ 1\\ -2\end{bmatrix}=\begin{bmatrix}-6\\ 4\\ -5\end{bmatrix},\quad x_1=-6, x_2=4, x_3=-5$$

Problem 2: Solve the system $AX = 0$

when $A = \begin{bmatrix} 1 & 0 & -2 & 3 \\ 0 & 2 & 2 & -4 \\ 1 & 2 & 0 & -1 \end{bmatrix}$ and Reduced Augmented Matrix $\begin{bmatrix} 1 & 0 & -2 & 3 & 0 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

1) (10pts) Put the solutions into vector form and name the free variables (parameters).

   From the RREF matrix we have that $\begin{cases} x_1 - 2x_3 + 3x_4 = 0 \\ x_2 + x_3 - 2x_4 = 0 \end{cases}$

   Using the leading ones as guides we solve for x_1 and x_2 to find

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2x_3 - 3x_4 \\ -x_3 + 2x_4 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -3 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

   and x_3 and x_4 are our parameters (or free variables)

2) (4pts)Give the dimension of the solution space (NullA) spanned by the vectors in part 1) and also give the larger space that contains this solution space.

   Since $v_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -3 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ is the basis for Null A, this subspace will have dimension 2 in the larger space of R^4.

3) (10pts) Determine if the vector $v = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \end{bmatrix}$ in this solution space? Show work.

Is v a linear combination of these base vectors? Can you find coefficients so that :

$$C_1 \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix} + C_2 \begin{bmatrix} -3 \\ 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \end{bmatrix}$$

Set up augmented matrix

$$\begin{bmatrix} 2 & -3 & 1 \\ -1 & 2 & -1 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix} \rightarrow RREF \rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{where } 0 \neq 1$$

Since system is inconsistent there is no solution and v is not in the solution space.

Problem 3: Find the unit vector associated with $v = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$.

Solution: (10pt)

Unit vector $u = \frac{v}{\|v\|} = \frac{1}{\sqrt{v \bullet v}} v = \frac{1}{\sqrt{16+3}} v = \begin{bmatrix} 4/5 \\ -3/5 \end{bmatrix}$

Problem 4: (12pts)

Find the value(s) of **k** so that $\cos\theta = 0$ when $u = \begin{bmatrix} k \\ k \\ 1 \end{bmatrix}, v = \begin{bmatrix} k \\ 5 \\ 6 \end{bmatrix}$. The angle θ is $\pi/2$.

$\text{Cos}\,\theta = \frac{v \bullet u}{\|v\|\|u\|}$ In order for $\text{Cos}\theta = 0$, $v \bullet u = 0$. If the dot product is zero then these two vectors are orthogonal and the angle will be 90 degrees.

In order for $v \bullet u = 0$, $k^2 + 5k + 6 = (k+3)(k+2) = 0$ and k = -2 or k = -3 will work.

Problem 5: (8pts) Given the matrices $A_{2x3}, B_{3x4}, C_{5x4}$ find m and n so that $(AB)C^T = F_{mxn}$ Show your reasoning.

$A_{2x3}B_{3x4}C^T_{4x5} = (AB)_{2x4}C^T_{4x5} = F_{2x5}$ so m = 2 and n = 5.

Problem 6: given the vectors $v_1 = \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$ verify that these vectors form a basis for R^3 using the **precise definition of linearly independent**.

(12pts)
By definition v_1, v_2, v_3 are linearly independent if $C_1v_1 + C_2v_2 + C_3v_3 = 0$ **only when** the coefficients $C_1 = C_2 = C_3 = 0$.
Set up an augmented matrix and solve for the coefficients.

$$\begin{bmatrix} 0 & 2 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix} \to \text{show work} \to \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$ to get that $C_1 = 0, C_2 = 0, C_3 = 0$

Problem 7: (10pts) Given $A_{p\times p}$ so that A^{-1} exist, answer the following by explaining the importance of A^{-1} in your conclusion.

a) What is the Rank of A? Why?
Since the inverse of A exist, A can be reduced the identity matrix of the same size. Each row will have a pivot point and there will be no zero rows. The Rank A = p.

b) Are the columns of A linearly independent? Why or Why not?
Since the inverse of A exist, A can be reduced the identity matrix of the same size. Every row has a pivot point which means every column also has a pivot point. The columns are Linearly independent.

Problem 8 : (10pts) Given the system AX = B and matrix A_{5x7} has Rank A = 4, answer the following and explain your conclusion.

a) What are the number of free variables? What is your reasoning?

   The RankA = 4 and the number of variables is 7. The rank indicates the number of non-zero rows left after A is reduced to RREF. Since there are less rows than variables, 7 – 4 will leave 3 free variables. (This also means there are 4 leading ones and three variables that will be parameters.)

b) Are the columns in A are linearly independent? Why or Why not?
   There are seven columns in A since there are seven variables. However, there are only 4 pivot points which means that three columns must be a linear combination of the other four and therefore the columns are not linear independent.