

Math 2114 Homework 3 Spring 2018

Show all work. (Points may be taken off if you do not follow the **Homework Requirements**)

Problem 1: Given the following system of equations:

$$\begin{cases} 3x_2 - 6x_3 + 6x_4 + 4x_5 = -5 \\ 3x_1 - 7x_2 + 8x_3 - 5x_4 + 8x_5 = 9 \\ 3x_1 - 9x_2 + 12x_3 - 9x_4 + 6x_5 = 15 \\ 6x_2 - 12x_3 + 12x_4 + 8x_5 = -10 \end{cases}$$

- Put into $AX = B$ form.
- Put in Reduced Row Echelon and determine the Rank of A
- Express the system as a linear combination of the column vectors
- Determine if the column vectors of A are linearly independent or dependent.
(You only need to think about work in part b, you do not need to show extra work)

Problem 2: Given the matrices below solve for a, b, c, d by creating a system of equations and solve using the augmented matrix.

$$\begin{bmatrix} a-b & b+c \\ 3d+c & 2a-4d \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ -4 & 2 \end{bmatrix}$$

Problem 3:

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 3 & 0 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 0 & -2 & 4 \\ -1 & 3 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 2 & -1 \\ 3 & -3 \end{bmatrix} \quad D = \begin{bmatrix} 1 & -1 \\ 4 & 6 \\ 2 & 1 \end{bmatrix}$$

Given the matrices above find the specified matrices (if possible) using matrix algebra. If not possible give the reason

- $2A - B$, 2) $A^T C$, 3) AC , 4) CB , 5) BD , 6) DB , 7) $D + A$

Problem 4:

Given the following matrices $A_{4 \times 5}$, $B_{4 \times 5}$, $C_{5 \times 2}$, $D_{4 \times 2}$, $E_{5 \times 4}$, determine if the following matrix expressions are defined. If the expression is defined then give the size of the resulting matrix.

a) AB , b) $AC + D$, c) $E(AC)$ d) $AE + B$ e) $E(A + B)$ f) $AB + B$

Problem 5: Find the following matrix product and show all work.

$$\left(\begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 3 & 4 & 2 & 1 \\ 1 & -1 & 3 & 2 \end{bmatrix} \right) \begin{bmatrix} -3 \\ 1 \\ -1 \\ 0 \end{bmatrix}$$

Problem 6: Set up a 5×5 matrix A with entries a_{ij} such that $a_{ij} = 4$ if $i = j$ and $a_{ij} = -2$ if $i < j$ and $a_{ij} = 7$ if $i > j$.

Problem 7: Determine the possible type of solution for each of the **augmented matrices** described below. Indicate if solution is unique or infinite with possible number of free variables or inconsistent.

- a) A is a 4×5 matrix with $\text{rank } A = 4$.
- b) A is a 4×4 matrix with $\text{rank } A = 1$.
- c) A is a 2×4 matrix with $\text{rank } A = 2$.
- d) A is a 4×3 matrix with $\text{rank } A = 2$.