

## Math 2114 Homework 1 Spring 2018

Follow **Homework requirements found on my website**. Put work on separate sheet of paper.

**Problem 1:** Given vectors  $v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ ,  $w = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ , find 1)  $v + w$ , 2)  $v - w$ , and 3)  $2v + w$ . Sketch the Parallelogram Rule that illustrates the results each operation.

### Problem 2:

Use Theorem 1.1 to justify each step given below to prove that  $u - (3u + v) + 2v + 2u = v$  For vectors  $u$  and  $v$  in  $\mathbb{R}^n$ . (**you may put your answers on this sheet for this problem**)

Proof: Consider the following:

$$\begin{aligned} u - (3u + v) + 2v + 2u &= \underline{\hspace{2cm}} \text{ given} \\ u + (-1)(3u + v) + 2v + 2u &= \underline{\hspace{2cm}} \text{ definition of scalar} \\ u + (-3)u + (-1)v + 2v + 2u &= \underline{\hspace{2cm}} \\ (-3)u + (-1)v + 2v + 2u + u &= \underline{\hspace{2cm}} \\ -3u + (-v + 2v) + (2u + u) &= \underline{\hspace{2cm}} \\ -3u + (v) + (3u) &= \underline{\hspace{2cm}} \\ v + (3u) + (-3u) &= \underline{\hspace{2cm}} \\ v + 0 &= \underline{\hspace{2cm}} \\ v &= \underline{\hspace{2cm}} \end{aligned}$$

**Problem 3:** Given the vectors  $v_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$ ,  $v_2 = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$ ,  $v_3 = \begin{bmatrix} 0 \\ -6 \\ 2 \end{bmatrix}$ ,

- Find the dot product  $v_1 \cdot v_2$ .
- Find the norm  $\|v_3\|$ .
- Find the unit vector associated with  $v_3$ .
- Find the linear combination  $u = 2v_1 - 4v_2 + 3v_3$
- Verify that  $\|v_1 + v_2\| < \|v_1\| + \|v_2\|$  when  $v_1 \cdot v_2 \neq 0$  ( Explain why.)

This problem is based on the Cauchy-Schwarz Inequality and the Triangle Inequality Found on page 22 and 23 in Poole.

- Verify that  $v_2$  is orthogonal to  $v_3$ .

- Find the angle  $\theta$  between  $v_1$  and  $v_2$  using  $\cos \theta = \frac{v_1 \cdot v_2}{\|v_1\| \|v_2\|}$