1. Believe it or not, the following function is constant on an interval \([a, b]\). Find that interval and the constant value of the function.

\[
f(x) = \sqrt{x + 2\sqrt{x} - 1} + \sqrt{x - 2\sqrt{x} - 1}.
\]

2. (VT Regional Math Contest) A walker and a jogger travel along the same straight line in the same direction. The walker walks at one meter per second, while the jogger runs at two meters per second. The jogger starts one meter in front of the walker. A dog starts with the walker, and then runs back and forth between the walker and the jogger with constant speed of three meters per second. Let \(f(n)\) meters denote the total distance travelled by the dog when it has returned to the walker for the \(n\)th time (so \(f(0) = 0\)). Find a formula for \(f(n)\).

3. (From Putnam 1942, problem A-3) Is the following series convergent or divergent?

\[
1 + \frac{\frac{19}{2}}{\frac{7}{3}} + \frac{2!}{3^2} \left(\frac{19}{7}\right)^2 + \frac{3!}{4^3} \left(\frac{19}{7}\right)^3 + \frac{4!}{5^4} \left(\frac{19}{7}\right)^4 + \cdots
\]

4. A spherical, 3–dimensional planet has center at \((0, 0)\) and radius 20. At any point of the surface of this planet, the temperature is \(T(x, y, z) = (x + y)^2 + (y - z)^2\) degrees. What is the average temperature of the surface of this planet?

5. Show that the equation

\[
n_1^4 + n_2^4 + \cdots + n_{14}^4 = 1599
\]

has no solutions in nonnegative integers. Hint: Think \(\mod 16\).