88403 MATH 5414 Robust Control Fall 2015; expected continuation to Spring 2016

Fall 2015 MW 2:30–3:45 pm RAND 120; expected continuation to Spring 2016

Instructor: Joseph A. Ball: joball@math.vt.edu, McB 554, office phone: 231-7080

Course Description: We will survey the following phases in the development of Robust (also called $H^\infty$) Control theory starting in the early 1980s:

1. The criticisms of George Zames in the early 1980s of the then fashionable but overly used LQG (Linear-Quadratic-Gaussian) approach to control. Whereas the LQG approach calls for every problem to have a statistical model and for the control design to minimize the expected value of an unwanted error signal, the deterministic robust $H^\infty$-control approach seeks a single controller which maintains stability and performance independent of which uncertainty from a whole admissible ball of uncertainties is actually present (worst-case design).

2. When formulated in frequency-domain terms, the simplest case of the problem turns out to be equivalent to a classical Nevanlinna-Pick interpolation problem (a surprise to Zames at the time). It is especially appropriate to talk about this now since we are approaching the centennial of the first paper on Nevanlinna-Pick interpolation (Pick’s first paper appeared in 1916). More realistic versions led to more sophisticated connections with function and operator theory (matrix-valued Nevanlinna-Pick interpolation, commutant lifting theory in operator theory).

3. State-space solutions: A major breakthrough appearing in 1989 was the elegant state-space solution of the $H^\infty$-problem due to Doyle-Francis-Glover-Khargonekar which involved a pair of Riccati equations with a coupling condition.

4. The 1990s saw a new state-space solution due to Apkarian-Gahinet for the suboptimal problem which involves Linear Matrix Inequalities (LMIs) rather than Riccati equations; the derivation of this solution is particularly elegant using only simple linear algebra. The LMI solution is particularly appealing due to recent advances to semi-definite programming making possible fast, accurate numerical solution of LMIs possible, at least for problems with state space dimension not too large. This LMI solution of the $H^\infty$-problem has the additional advantage that it extends almost seamlessly to multidimensional systems (where the evolution in the state-space model is with respect to both a time variable and a space variable). More recently there have been additional advances using sums-of-squares techniques which also lead quickly to practical LMI problems.

5. More refined versions of the robust-control theory ask for robustness with respect to structured uncertainty (as opposed to the unstructured case where the admissible uncertainty is modeled by a matrix ball); it turns out that models for structured uncertainty are closely connected with the multidimensional systems mentioned above and with several-variable complex function theory (in commuting or noncommuting variables).
The course will provide a leisurely survey of these developments. The emphasis will be on engineering motivation, mathematical ideas and the flavor (but not the actual coding up) of numerical implementation.

**Expectations:** Expectations for a grade of A in the course are minimal: regular attendance. There may be occasional suggested homework assignments but these are optional. For those already involved in their own dissertation research, a private oral report on the project (with possible followup report to the whole class if of sufficient interest and relevance for the course) will substitute for doing homework problems.

This course is an update of a similar Topics Course offered in 2009-2010.

Prerequisites: some real analysis, complex analysis, linear algebra. Some background in control theory, functional analysis, and operator theory would be helpful but is not required.


