LES of Gravity Currents
The Filter Radius and the Van Driest Damping

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1 Introduction

We start by presenting some general remarks concerning two important parameters in the LES of gravity currents: (i) $\delta$, the radius of the spatial filter used in LES to describe the large structures, and (ii) the Van Driest damping (VDD), the function which is generally used with the eddy-viscosity LES models, such as Smagorinsky.

Before we analyze and describe our implementation for each of them separately, we need to highlight their similarity: both $\delta$ and the VDD aim at reducing the contribution of the eddy-viscosity model near the solid boundaries. Otherwise, the eddy-viscosity LES models become unstable in numerical simulations and yield unphysical results, as documented in many studies.

2 The filter radius - $\delta$

2.1 Popular choice of $\delta$ in the LES literature

The most popular choice of $\delta$ in the LES literature is

$$\delta = (\Delta x \Delta y \Delta z)^{1/3},$$

where $\Delta x$, $\Delta y$, and $\Delta z$ are the components of $\delta$ in the $x$, $y$, and $z$ directions, respectively. A very common choice for computing them is $\Delta x \approx 2h_x$, $\Delta y \approx 2h_y$, $\Delta z \approx 2h_z$, where $h_x$, $h_y$, and $h_z$ are the meshsizes in the $x$, $y$, and $z$ directions, respectively.

It is very common to take constant $h_x$ and $h_z$ and choose $h_y$ so that it goes to 0 near the solid boundary.

2.2 Spectral element case

The spectral element case is a little bit more delicate. The reason is that the spacing between the meshpoints is not uniform. Specifically, the meshpoints are clustered near the interface of the spectral elements (bricks). Thus, one needs to define $h_x$ and $h_z$.

In my papers with Paul, we took these to be the minimum spacing between two consecutive meshpoints in a spectral element. But we have not investigated in detail the effect of our choice on the results, although we acknowledged that this effect could be significant.

Either way, spectral elements or not, one needs to make a choice. Moreover, the effect of this choice in LES is not clearly understood. Basically, people do what the others did before them...
2.3 Our implementation of $\delta$

Initially, I tried to use a parabola in the vertical ($y$) direction, but I don’t think this is the right approach, since we really need the LES model have a significant contribution near the bottom, where all the interesting motion takes place. We need something more aggressive.

Thus, a better idea seems to be the arctan function. I chose the arctan function so that the radius of the filter ($\delta$) goes fast to 0 near the bottom, and then it reaches a prescribed value away from the bottom. Thus, $\delta(x, y, z)$, the filter radius at the point with coordinates $(x, y, z)$, is given by

$$
\delta = 0.01 \ast \frac{2.0}{\pi} \arctan(100.0 \ast (y - \bar{y}))
$$

where $\bar{y}$ is the vertical coordinate of the bottom corresponding to $(x, z)$. From (2), we notice that

- $\delta = 0$ on the bottom of the slope, i.e. when $y = \bar{y}$;
- $\delta \approx 0.01$ on top of the computational domain, i.e. when $y = 1.0$;

Notice that the maximum value of $\delta$ is 0.01, which is one fifth of the height of one spectral element. This is somehow ad-hoc, and we need to experiment with it.

We have not so far included a $x$- and $z$-behavior, and this is something we need to investigate as well. My experience with $\delta$ is that, although the formula we use in calculating it does matter, it probably does not account for the changes in the NO-LES vs. LES we are looking for. In other words, it is significant, but not that important. But this is definitely something we should look into eventually.

![Figure 1: Sketch of $\delta$ as a function of $y$](image-url)
3 The Van Driest Damping

The Van Driest damping (VDD) function has the same goal as $\delta$: it tries to make the LES contribution go to 0 near the solid boundary. Otherwise, the numerical computations are very unstable, as reported in many studies. The VDD, though, is a little bit more subtle: it decreases the Smagorinsky contribution so that the average of the streamwise (in the $x$-direction) velocity follows the boundary layer theory (logarithmic law of the wall).

My main concern about the VDD, however, is that our flow is transient. I am not exactly sure that our flow obeys the boundary layer theory, which makes the whole VDD approach at least questionable.

But let’s assume that the theory supporting the VDD were true. The implementation of VDD would take some effort, since we would need to compute $u_\tau$, the wall shear velocity, based on averages of the wall shear stress $\tau$.

The van Driest scaling reads

$$C_V = C_S(x) = \left[ C_s \delta \left( 1 - e^{-y^+/A} \right) \right]^2,$$

where $C_s = 0.17$ is Lilly’s constant and $y^+$ is the non-dimensional distance from the wall

$$y^+ = \frac{u_\tau y}{\nu},$$

which determines the relative importance of viscous and turbulent phenomena. In (4), $u_\tau$ is the wall shear velocity, and $A = 25$ is the van Driest constant.

The van Driest scaling (3) improves the performance of the model in predicting statistics of turbulent flow in simple geometries, where boundary layer theory holds (e.g., flow past a flat plate and pipe flow).