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Approximation of the International Space Station 1R and 12A Models

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Abstract

In this note we examine the model reduction problem for the two structural systems namely stage 1R and stage 12A of the International Space Station (ISS). Several model reduction algorithms are applied and the results are compared.

1 Introduction

The International Space Station (ISS) is a complex structure composed of many stages. It is estimated that more than 40 Shuttle flights will be required to complete the assembly. Also, each stage can be modeled by means of approximately 1000 state variables. As more stages are added, the complexity increases at each step. For ISS, controllers will be needed for many reasons; such as to reduce the oscillatory motions, or to fix the orientation of the space station with respect to some desired direction. Generically a controller of a given plant has the same complexity as the plant. In this case, since these controllers have to be implemented onboard, low complexity is a requirement. Hence, model reduction of the ISS stages is essential for obtaining reduced order controllers.

The Charles Stark Draper Laboratory uses the so-called Model Gain Factor (MGF) method [4] for model reduction. Basically, this is a multi-variable generalization of the SISO modal approximation method which weighs the flexibility modes by frequency. In this paper, we investigate the model reduction of the two stages, (1) stage 1R and (2) stage 12A of the ISS. The following methods are used and compared: (1) Balanced reduction (**BR**) [5]; (2) approximate balanced reduction (**ABR**) [1], [3]; (3) weighted balanced reduction (**WBR**) [2]; (4) the Arnoldi (**AP**) and Lanczos (**LP**)procedures [1]; (5) Model Gain Factor (**MGF**) method. We also use Smith [6] method to compute the approximate low-rank Grammians. Then **BR** and **ABR** are applied using the low-rank Grammians. Here thereafter we will call these approximants *Smith-balanced* (**SBR**) and *Smith-approximate balanced reduction* (**SABR**) models respectively.

2 Stage 1R

This is a model of stage 1R (Russian Service Module) of the ISS. It has 270 states, 3 inputs and 3 outputs. We approximate the system with reduced models of order 26. We apply **BR** , **ABR** , **WBR** ; and Smith methods **SBR** and **SABR** . For **WBR** , both input $W_i(s)$ and output $W_o(s)$ weights are included with $W_i(s) = W_o(s)$. Three types of weightings are considered: (1) The transfer function of each input-output channel of $W_i(s)$ and $W_o(s)$ is a band pass filter over the frequency range $s = [0.5 - 100]$ rad/sec, (2) only diagonal weighting is considered, (3) the transfer function of each input-output channel is a band pass filter over $s = [0.5 - 10]$ rad/sec frequency range. Since **BR** and **ABR** yields almost the same results, we depict the results only for **BR** . The same also holds true for **SBR** and **SABR** ; and we depict only the former. Figure 1 shows the largest singular value σ_{max} of the frequency response of the error systems. Although MGF method is better for low frequencies, **BR** and **ABR** have slightly less error at moderate and high frequencies. **SBR** and **SABR** methods are also comparable with the other methods especially at the moderate frequencies. Indeed for high frequencies they are the best approximants. As seen from Figure 1, among the **WBR** models, the second approximant is the best. Indeed, this approximant gives almost the same result as the balancing. The first weighted approximant has problem for $s = [10 - 100]$ rad/sec. On the other hand, the third weighted balanced approximant matches the full order model very well for low frequencies but for high frequencies it deviates a lot as expected. Table 1 lists the relative H_∞ norms of the error systems. As the table shows, **BR** and **ABR** have the lowest error norm. **MGF** method has a slightly higher error norm. Both Smith approximants yielded high error norms. The worst among all is the third weighted balanced reduction approximant.

BR	SBR	MGF
5.71×10^{-3}	2.44×10^{-2}	5.75×10^{-3}
WBR -1	**WBR -2**	**WBR -3**
2.73×10^{-2}	5.71×10^{-3}	8.04×10^{-2}

Table 1: Relative H_∞ error norms for model 1R

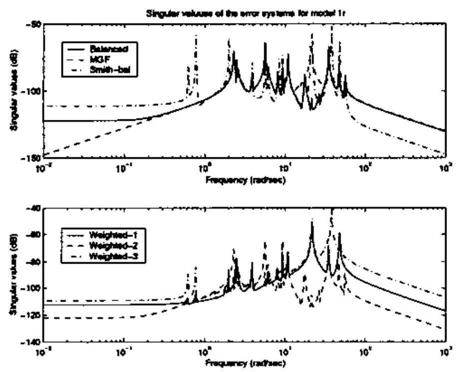

Figure 1: σ_{max} plot of the error systems of model 1R

3 Stage 12A

This is a model of stage 12A of the ISS. It has 1412 states, 3 inputs and 3 outputs. We will consider only a SISO subsystem of this model corresponding to the first input and the first output. The system is approximated with reduced models of order 226. We apply **BR** , **ABR** ; **AP** and **LP** ; and Smith methods. Again **BR** and **ABR** yield almost the same results and we depict the results only for **BR** . The same also holds true for **SBR** and **SABR** . We plot the largest singular value σ_{max} of the frequency response of the error systems in Figure 2. Clearly, the **LP** and **AP** outperform all the other methods around $s = 0$ and $s = \infty$. For moderate frequencies all the methods are comparable except for the **LP** which has high error in this range. Like the model 1R example, while **MGF** method is better than **BR** around $s = 0$, **BR** is better for high frequencies. The relative H_∞ norms of the error systems are listed in Table 2. As the table shows, **BR** and **ABR** have the lowest error norm. The **MGF** error in this case is much higher than that of **BR** due to the deviation around $s = 5$ rad/sec. Despite being the best approximants around $s = 0$ and $s = \infty$, the **LP** and **AP** yield also high error norms due to being local in nature. The worst among all is the **LP** . Smith methods **SBR** and **SABR** lies between **MGF** method and **AP** .

BR	SBR	MGF
1.43×10^{-3}	3.74×10^{-2}	2.05×10^{-2}
AP	**LP**	
4.11×10^{-2}	3.49×10^{-1}	

Table 2: Relative H_∞ error norms for model 12A

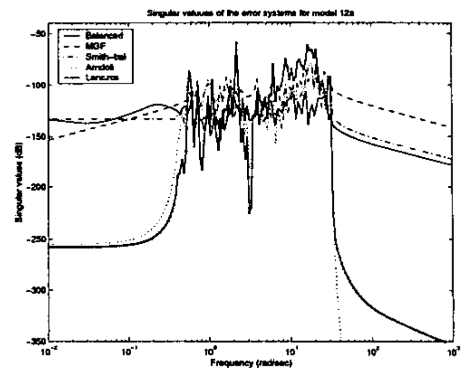

Figure 2: σ_{max} plot of the error systems of model 12A

4 Conclusions

In this note we considered the model reduction problem for the two structural systems, stage 1R and stage 12A of the International Space Station. We apply several model reduction problems and compare the approximants with those obtained by the model gain factor method. We see that while **MGF** method is better than **BR** and **ABR** for low frequencies, **BR** and **ABR** are preferable for high frequencies. Also, in both cases, **BR** and **ABR** yield the lowest error norms. Moreover, Smith method yields quite satisfactory reduced models. Due to being local in nature **LP** and **AP** yield high error norms although they are the best for high and low frequencies for the model 12A. **WBR** also yields satisfactory approximants when the weighting is properly chosen.

References

[1] A.C. Antoulas, D.C. Sorensen, and S. Gugercin, *A survey of model reduction methods for large-scale systems*, Structured Matrices in Operator Theory, Numerical Analysis, Control, Signal and Image Processing, Contemporary Mathematics, AMS publications, 2001.

[2] D. Enns, *Model reduction with balanced realizations: An error bound and a frequency weighted generalization*, in the Proceedings of 23^{rd} CDC, 1984.

[3] S. Gugercin and A.C. Antoulas, *A comparative of 7 algorithms for model reduction*, in the Proceedings of 39^{th} CDC, December 2000.

[4] S. Gugercin, A.C. Antoulas and N. Bedrossian, *Approximation of the International Space Station 1R and 12A Flex Models*, Techinal Report, August 2001.

[5] B. C. Moore, *Principal Component Analysis in Linear System: Controllability, Observability and Model Reduction*, IEEE Trans. on Auto. Contr., AC-26:17-32, 1981.

[6] R. A. Smith, *Matrix Equation, $XA + BX = C$*, SIAM J. Appl. Math, 16: 198-201, 1968.