Approximation of the International Space Station 1R and 12A Models

S. Gugercin and A.C. Antoulas
Department of Electrical and Computer Eng.
Rice University, Houston, Texas 77251-1892, USA
{serkan,aca}@rice.edu

N. Bedrossian
The Charles Stark Draper Lab. Inc.
Houston, TX 77058, USA
naz@jsc.draper.com

Abstract

In this note we examine the model reduction problem for the two structural systems namely stage 1R and stage 12A of the International Space Station (ISS). Several model reduction algorithms are applied and the results are compared.

1 Introduction

The International Space Station (ISS) is a complex structure composed of many stages. It is estimated that more than 40 Shuttle flights will be required to complete the assembly. Also, each stage can be modeled by means of approximately 1000 state variables. As more stages are added, the complexity increases at each step. For ISS, controllers will be needed for many reasons; such as to reduce the oscillatory motions, or to fix the orientation of the space station with respect to some desired direction. Generically a controller of a given plant has the same complexity as the plant. In this case, since these controllers have to be implemented onboard, low complexity is a requirement. Hence, model reduction of the ISS stages is essential for obtaining reduced order controllers.

The Charles Stark Draper Laboratory uses the so-called Model Gain Factor (MGF) method [4] for model reduction. Basically, this is a multi-variable generalization of the SISO modal approximation method which weighs the flexibility modes by frequency. In this paper, we investigate the model reduction of the two stages, (1) stage 1R and (2) stage 12A of the ISS. The following methods are used and compared: (1) Balanced reduction (BR) [5]; (2) approximate balanced reduction (ABR) [1], [3]; (3) weighted balanced reduction (WBR) [2]; (4) the Arnoldi (AP) and Lanczos (LP) procedures [1]; (5) Model Gain Factor (MGF) method. We also use Smith's method [6] to compute the approximate low-rank Grammians. Then BR and ABR are applied using the low-rank Grammians. Hereafter we will call these approximants Smith-balanced (SBR) and Smith-approximate balanced re-

duction (SABR) models respectively.

2 Stage 1R

This is a model of stage 1R (Russian Service Module) of the ISS. It has 270 states, 3 inputs and 3 outputs. We approximate the system with reduced models of order 26. We apply BR, ABR, WBR; and Smith methods SBR and SABR. For WBR, both input $W_i(s)$ and output $W_o(s)$ weights are included with $W_i(s) = W_o(s)$. Three types of weightings are considered: (1) The transfer function of each input-output channel of $W_i(s)$ and $W_o(s)$ is a band pass filter over the frequency range $s = [0.5 - 100]$ rad/sec, (2) only diagonal weighting is considered, (3) the transfer function of each input-output channel is a band pass filter over $s = [0.5 - 10]$ rad/sec frequency range. Since BR and ABR yields almost the same results, we depict the results only for BR. The same also holds true for SBR and SABR; and we depict only the former. Figure 1 shows the largest singular value $\sigma_{max}$ of the frequency response of the error systems. Although MGF method is better for low frequencies, BR and ABR have slightly less error at moderate and high frequencies. SBR and SABR methods are also comparable with the other methods especially at the moderate frequencies. Indeed for high frequencies they are the best approximants. As seen from Figure 1, among the WBR models, the second approximant is the best. Indeed, this approximant gives almost the same result as the balancing. The first weighted approximant has problem for $s = [10 - 100]$ rad/sec. On the other hand, the third weighted balanced approximant matches the full order model very well for low frequencies but for high frequencies it deviates a lot as expected. Table 1 lists the relative $H_\infty$ norms of the error systems. As the table shows, BR and ABR have the lowest error norm. MGF method has a slightly higher error norm. Both Smith approximants yielded high error norms. The worst among all is the third weighted balanced reduction approximant.
Table 1: Relative $H_\infty$ error norms for model 1R

<table>
<thead>
<tr>
<th>BR</th>
<th>SBR</th>
<th>MGF</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.71 x 10^{-3}</td>
<td>2.44 x 10^{-2}</td>
<td>5.75 x 10^{-3}</td>
</tr>
<tr>
<td>WBR -1</td>
<td>WBR -2</td>
<td>WBR -3</td>
</tr>
<tr>
<td>2.73 x 10^{-2}</td>
<td>5.71 x 10^{-3}</td>
<td>8.04 x 10^{-2}</td>
</tr>
</tbody>
</table>

Table 2: Relative $H_\infty$ error norms for model 12A

<table>
<thead>
<tr>
<th>BR</th>
<th>SBR</th>
<th>MGF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.43 x 10^{-3}</td>
<td>3.74 x 10^{-2}</td>
<td>2.05 x 10^{-2}</td>
</tr>
<tr>
<td>AP</td>
<td>LP</td>
<td></td>
</tr>
<tr>
<td>4.11 x 10^{-2}</td>
<td>3.49 x 10^{-1}</td>
<td></td>
</tr>
</tbody>
</table>

Figure 1: $\sigma_{max}$ plot of the error systems of model 1R

3 Stage 12A

This is a model of stage 12A of the ISS. It has 1412 states, 3 inputs and 3 outputs. We will consider only a SISO subsystem of this model corresponding to the first input and the first output. The system is approximated with reduced models of order 226. We apply BR, ABR, AP and LP; and Smith methods. Again BR and ABR yield almost the same results and we depict the results only for BR. The same also holds true for SBR and SABR. We plot the largest singular value $\sigma_{max}$ of the frequency response of the error systems in Figure 2. Clearly, the LP and AP outperform all the other methods around $s = 0$ and $s = \infty$. For moderate frequencies, all the methods are comparable except for the LP which has high error in this range. Like the model 1R example, while MGF method is better than BR around $s = 5$ rad/sec. Despite being the best approximants around $s = 0$ and $s = \infty$, the LP and AP yield also high error norms due to being local in nature. The worst among all is the LP. Smith methods SBR and SABR lies between MGF method and AP.

![Figure 2: $\sigma_{max}$ plot of the error systems of model 12A](image)

4 Conclusions

In this note we considered the model reduction problem for the two structural systems, stage 1R and stage 12A of the International Space Station. We apply several model reduction problems and compare the approximants with those obtained by the model gain factor method. We see that while MGF method is better than BR and ABR for low frequencies, BR and ABR are preferable for high frequencies. Also, in both cases, BR and ABR yield the lowest error norms. Moreover, Smith method yields quite satisfactory reduced models. Due to being local in nature LP and AP yield high error norms although they are the best for high and low frequencies for the model 12A. WBR also yields satisfactory approximants when the weighting is properly chosen.

References


