Form A

Math 2214 Common Part of Final Exam December 14, 1998

<u>**INSTRUCTIONS**</u>: Please enter your NAME, ID NUMBER, FORM designation, and INDEX NUMBER on your op-scan sheet. The index number should be written in the upper right-hand box labeled “Course”. In the box labeled “Form”, write the appropriate test form letter **A**. Darken the appropriate circles below your ID number and Form designation. **Use a #2 pencil**.

Mark your answers to the test questions in rows 1-14 of the op-scan sheet. You have 1 hour to complete this part of the final exam. Your score on this part of the final exam will be the number of correct answers. Please turn in the op-scan sheet with your answers <u>and this question sheet</u> at the end of this part of the final exam.

1. The solution of the initial value problem

$$y' + 7y = 0, \quad y(0) = 12 \quad \text{is}$$

(1) $12e^{7t}$ (2) $12e^{-7t}$ (3) $7e^{12t}$ (4) $7e^{-12t}$

2. Suppose that the third order differential equation $e^t u''' + (\cos t)u' + e^t u = 0$ is equivalent to the system $\dfrac{d\mathbf{x}}{dt} = A\mathbf{x}$. Find A.

(1) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -e^{t} & -\cos t & 0 \end{pmatrix}$ (2) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -e^{-t}\cos t & -1 & 0 \end{pmatrix}$

(3) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -e^{-t}\cos t & 0 \end{pmatrix}$ (4) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -e^{-t}\cos t & -1 \end{pmatrix}$

3. Let $y(t)$ be the solution of the initial value problem

$$y' = \sqrt{1 - y^2}, \quad y(0) = 0.$$

By examining the direction field or otherwise, determine which one of the following equals $y(4\pi)$.

(1) -1 (2) 0 (3) 1 (4) 2

4. Consider the differential equations

(NH) $y'' + p(x)y' + q(x)y = g(x)$

and

(H) $y'' + p(x)y' + q(x)y = 0$.

If $y_1(x) = x$ and $y_2(x) = x^{-1}$ are solutions of (H), and $\phi(x) = x^2$ is a solution of (NH), then the solution of the initial value problem

$$y'' + p(x)y' + q(x)y = g(x), \quad y(1) = 0, \quad y'(1) = 1$$

is

(1) $\frac{x}{2} - \frac{1}{2x}$ (2) $-x + x^2$

(3) $-\frac{5x}{2} + \frac{1}{2x} + 2x^2$ (4) $-\frac{1}{3x} + \frac{x^2}{3}$

5. For which values of the real constant α does the solution pair

$$y_1(x) = \alpha e^x + e^{-x}, \quad y_2(x) = e^x + \alpha e^{-x}$$

form a fundamental set of solutions for the differential equation $y'' - y = 0$?

(1) Every α

(2) Every α except $\alpha = 1$

(3) Every α except $\alpha = 1$ and $\alpha = -1$

(4) Only for $\alpha = 1$ and $\alpha = -1$

6. Let A be a 2x2 constant matrix with real entries. Suppose that $\lambda = 1$ is a repeated eigenvalue of A with eigenvector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Assuming that $(A - I)\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, find the solution of the following initial value problem:

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

(1) $\begin{pmatrix} e^t \\ 2e^t \end{pmatrix}$ (2) $\begin{pmatrix} e^t + te^t \\ 2e^t \end{pmatrix}$

(3) $\begin{pmatrix} e^t + te^t \\ 2e^t + te^t \end{pmatrix}$ (4) $\begin{pmatrix} e^t + te^t \\ 2e^t + 2te^t \end{pmatrix}$

7. For the initial value problem

$$y' = t^2 - y^2, \quad y(2) = 3,$$

use one step of the Euler method to calculate an approximation of $y(2.1)$.

(1) -.5 (2) 2.5 (3) 2.9 (4) 3.1

8. The general solution of the equation

$$y''' + 3y'' - 4y = 0$$

is

(1) $Ae^{x} + Be^{-2x} + Cxe^{-2x}$
(2) $Ae^{x} + Be^{-2x}$
(3) $Ae^{x} + B\cos 2x + C\sin 2x$
(4) $Ae^{-x} + Be^{-2x} + Cxe^{-2x}$

9. Newton's Law of Cooling says

$$\frac{d\theta}{dt} = -k(\theta - T),$$

where $\theta(t)$ is the temperature (˚F) of a given object at time t (in minutes), T is the temperature of the surrounding environment, and k is a constant of proportionality.

A thermometer reads 60˚ F. It is taken outdoors, where the temperature is 0˚ F. One minute later, the thermometer reading is 50˚ F. Then the temperature reading 2 minutes after the thermometer was taken outdoors is

(1) $\dfrac{60}{2\ln(6/5)}$ (2) $\dfrac{60}{[\ln(6/5)]^2}$ (3) 40 (4) $\dfrac{125}{3}$

10. If the method of undetermined coefficients is used to find a particular solution y_p to the differential equation

$$y'' - 2y' + y = 2xe^{-x} + x,$$

then the assumed form of y_p should be

(1) $y_p = Ax^2e^x + Bxe^x + Cx + D$
(2) $y_p = Ax\,e^{-x} + Be^{-x} + Cx + D$
(3) $y_p = Axe^{-x} + Bx$
(4) $y_p = Axe^x + Be^x + Cx + D$

11. Let A be a 2x2 constant matrix with real entries. Suppose that A has eigenvalue $1 + 2i$ with eigenvector $\begin{pmatrix} 2 \\ i \end{pmatrix}$. Find the solution of the initial value problem

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

(1) $e^t\begin{pmatrix} \cos(2t) + \frac{1}{3}\sin(2t) \\ 2\cos(2t) - \sin(2t) \end{pmatrix}$ (2) $e^t\begin{pmatrix} \cos(2t) + 4\sin(2t) \\ 2\cos(2t) - \frac{1}{2}\sin(2t) \end{pmatrix}$

(3) $e^t\begin{pmatrix} \cos(2t) + \frac{1}{4}\sin(2t) \\ 2\cos(2t) + \sin(2t) \end{pmatrix}$ (4) $e^t\begin{pmatrix} \cos(2t) + \frac{1}{2}\sin(2t) \\ 2\cos(2t) - \frac{1}{4}\sin(2t) \end{pmatrix}$

12. Find the general solution of the DE $\frac{dy}{dt} + y = \cos(3t)$. (The symbols C, C_1 and C_2 denote arbitrary constants.) Hint: Examine the four choices.

(1) $C_1 e^{-t} + C_2 \sin(3t)$ (2) $\frac{1}{3} e^{-t} + C_1 \cos(3t) + C_2 \sin(3t)$

(3) $Ce^{t} + \frac{3}{10}\sin(3t) + \frac{1}{10}\cos(3t)$ (4) $Ce^{-t} + \frac{3}{10}\sin(3t) + \frac{1}{10}\cos(3t)$

13. If $y(t)$ is the solution of the initial value problem

$$y' - \frac{1}{t} y = 8t^2, \qquad y(1) = 8,$$

then $y(2)$ is equal to

(1) 19 (2) 20 (3) 38 (4) 40

14. Given that A is a 2x2 constant matrix with real entries, which of the following functions can be a solution of the system $\frac{d\mathbf{x}}{dt} = A\mathbf{x}$?

(1) $\begin{pmatrix} e^{t} - e^{3t} \\ -e^{t} + e^{2t} \end{pmatrix}$ (2) $\begin{pmatrix} 2e^{t} \\ te^{t} + e^{2t} \end{pmatrix}$

(3) $\begin{pmatrix} -e^{t} - e^{2t} \\ 5e^{t} + 5e^{2t} \end{pmatrix}$ (4) $\begin{pmatrix} e^{t} - e^{2t} \\ e^{t} + e^{2t} \end{pmatrix}$