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Exam 3; Math 4426; Due: 5-06-08

1. Let $x_k = \frac{k\pi}{4}$, and let $b_k^2(x)$ be the quadratic basis spline with support on the interval $[x_{k-1},\, x_{k+2}]$ and such that $b_k^2(x_k) = 4,\ k = -1, 0, 1, 2, 3, 4$. Find coefficients $c_k,\ k = -1, 0, 1, 2, 3, 4$, such that, with

$$s(x) = \sum_{k=-1}^{4} c_k\, b_k^2(x), \text{ we have } s(x_k) = \sin(\pi\, x_k),\ k = 0, 1, 2, 3, 4,\ s^{'}(x_0) = -s^{'}(x_4).$$

With $s(x)$ thus computed, compare $s\left(\frac{1}{3}\right)$ with $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

2. Let the points x_k, the basis splines $b_k(x)$ and the computed coefficients c_k be as in Problem 1. We want to estimate the unique $\hat{x} \in (0,\, \pi]$ such that $\sin\hat{x} = \frac{\hat{x}}{2}$. First find the interval $[x_{k-1},\, x_k]$ in which $s(x) - \frac{x}{2}$ changes sign and then solve the quadratic equation $s(x) - \frac{x}{2} = 0$, $s(x)$ as defined in that interval, to obtain an estimate $\tilde{x}$ for $\hat{x}$.

3. Let $x_k = k$ for all integers k (thus $h = 1$). Noting that we may then take

$$b_0^1(x) = \begin{cases} x + 1, & -1 \le x \le 0, \\ 1 - x, & 0 \le x \le 1, \\ 0 & \text{otherwise}, \end{cases}$$

verify that, if we normalize the cubic splines $b_k^3(x)$ to have maximum value 1 at x_k, we have the convolution relation

$$b_0^3(x) = \left(b_0^1 * b_0^1\right)(x), \qquad (f * g)(x) \equiv \int_{-\infty}^{\infty} f(\xi)\, g(x - \xi)\, d\xi.$$

4. With $h = x_k - x_{k-1} = .25$, use cubic splines to find, via collocation, an approximate solution $y(x)$ on $[0,\, 1]$ of the two point boundary value problem

$$\frac{d^2y}{dx^2} - y(x) + b_2^3(x) = 0, \qquad \frac{dy}{dx}(1) + 2\,y(1) = 0, \qquad \frac{dy}{dx}(0) - y(0) = 0.$$

Plot $y(x)$ so obtained with x increments of length .05 on $[0,\, 1]$. (Scale $b_2^3(x)$ to have maximum value 4.)

5. Let T be the triangle with vertices $P = (-1,\, 0),\ Q = (1,\, 0),\ R = (0,\, 2)$. Assume $u(x,\, y)$ is a linear function of x and y on T with vertex values $u(P) = 1,\ u(Q) = 2,\ u(R) = 1.5$. Compute the following:

$$u(-.5,\, .5), \qquad \frac{\partial u}{\partial x}(x,\, y),\ \frac{\partial u}{\partial y}(x,\, y),\ (x,\, y) \in T, \qquad \iint_T u(x,\, y)\, dx\, dy.$$