Use this sheet as a cover sheet for your exam paper. Staple, please.

Name ________________________________

Exam 2; Math 4426; Due: 4-08-08

1. Consider the heat equation $\frac{\partial u}{\partial t} - \Delta u = 0$ in the unit disk $0 \leq r \leq 1$, $0 \leq \theta \leq 2\pi$. Find the solution $u(r, \theta, t)$ which satisfies $u(1, \theta, t) \equiv 0$, $u(r, \theta, 0) = 5\, J_3(\omega_{32} r) \cos 3\theta - 3\, J_5(\omega_{53} r) \sin 5\theta$.

2. Compute $\int_0^1 r^2 J_0(r)\, dr$ numerically with an error $< 10^{-4}$ by multiplying the power series for $J_0(r)$ by r^2 to get a power series for $r^2 J_0(r)$ and integrate term by term to get a numerical series for the integral.

3. Let $u(r, \theta, t)$ be the solution of the heat equation $\frac{\partial u}{\partial t} - \Delta u = 0$ in the unit disk in R^2 with $u(r, \theta, 0) = 0$ and $u(1, \theta, t) \equiv \cos\theta$. Explain in detail how to compute $u(r, \theta, t)$ using the Bessel functions $J_1(\omega_{1j} r)$. Hint: let $u(r, \theta, t) = v(r, \theta, t) + w(r, \theta)$ where $w(r, \theta)$ solves the Dirichlet problem $\Delta w = 0$, $w(1, \theta) = \cos\theta$, substitute this expression into the heat equation and the initial condition and solve for $v(r, \theta, t)$.

4. Use the formula on page 2 of the notes on "The Laplacian in R^3; Spherical Harmonics" to compute $P_4^2(s)$ explicitly and, for that case, verify the proof following that formula to show that $P_4^2(s)$ is a solution of the associated Legendre equation of order $m = 2$. Then express $P_4^2(\cos\phi)$ in terms of trigonometric functions $\cos k\phi$, $\sin k\phi$ (eliminating powers $(\cos\phi)^j$, $j > 1$).

5. Let n be a fixed positive integer. Describe all functions $f(\theta, \phi)$ for which the solution of Laplace's equation $\Delta u = 0$ in $\mathcal{B}_a$, the ball of radius $a > 0$ in R^3, satisfying the boundary condition $u(a, \theta, \phi) = f(\theta, \phi)$, takes the form

$$u(r, \theta, \phi) = \left(\frac{r}{a}\right)^n f(\theta, \phi).$$

List these functions $f(\theta, \phi)$ explicitly for $n = 4$.