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Exam 2: Math 4245; Due: Tuesday, Nov. 10, 2009

1. Use the Laplace transform to find the solution of

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = H(t-1) - H(t-2), \quad y(0) = 0, \quad \frac{dy}{dt}(0) = 1.$$

2. Use the Laplace transform convolution property together with the residue method of partial fractions decomposition to find the variation of parameters formula for solution of

$$\frac{d^3y}{dt^3} - 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} - 6y = f(t), \quad y(0) = \frac{dy}{dt}(0) = \frac{d^2y}{dt^2}(0) = 0,$$

for a general function $f(t)$. Then find $y(t)$ for the particular case $f(t) = \sin t$.

3. Use the Laplace transform to find a particular vector solution of the linear inhomogeneous system

$$\frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \cos 2t \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

4. **a)** Find two linearly independent solutions of the two-dimensional vector system

$$\frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -2\,y_1/t + y_2 \\ 3\,y_2/t \end{pmatrix}.$$

and verify their linear independence. Use this result to construct the fundamental solution matrix $\mathbf{Y}(t,\, 0)$ which is equal to the identity matrix at $t = 1$.

b) Use the result of a) to find the solution of the system satisfying

$$y_1(1) = -1, \quad y_2(1) = 1.$$

5. **a)** Find three linearly independent solutions, and a fundamental matrix solution, for the system

$$\frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & -2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}.$$

b) Let $\mathbf{A}$ be the 3×3 matrix in part **a)**. Compute the matrix valued function $e^{\mathbf{A}t}$ i) using the fundamental matrix solution constructed in **a)**, and ii) using the Laplace transform.

6. Construct a 3 - dimensional, first order, system equivalent to the third order scalar equation

$$\frac{d^3y}{dt^3} - y^3 = 1.$$