Numerical Integration: The Trapezoidal and Simpson Rules - Konaté

a function f is defined on an interval $]a, b[$, and is assumed to be enough regular.

1. The Trapezoidal Rule: We divide $[a, b]$ in n subintervals such that:
$a = x_1 < x_2 < x_3 < \cdots < x_n < x_{n+1} = b$.
The Trapezoidal Rule states:

$$\int_a^b f(x)dx = \frac{h}{2}\left[f(x_1) + 2\sum_{i=2}^{n} f(x_i) + f(x_{n+1})\right].$$

Where

$$h = \frac{b-a}{n}.$$

The precision $E(h)$ is such that

$$E(h) \le \frac{h^3}{12}|f''(x)|.$$

1. The Simpson Rule: this time around, $[a, b]$ is divided into an even number, say 2n subintervals such that:
$a = x_1 < x_2 < x_3 < \cdots < x_{2n} < x_{2n+1} = b$.
The Simpson basic Rule states:

$$\int_a^b f(x)dx = \frac{h}{3}\left[f(x_1) + 2\sum_{i=1}^{n-1} f(x_{2i+1}) + 4\sum_{i=1}^{n} f(x_{2i}) + f(x_{2n+1})\right].$$

Where

$$h = \frac{b-a}{n}.$$

The precision $E(h)$ is such that

$$E(h) \le \frac{h^5}{90}|f^{(4)}(x)|.$$
