MATH4564 - Review For Test No3 - Fall 03 - Konaté

Notice: Show your work. A right answer with a bad reasoning will be considered as wrong. **Calculators allowed**. The Table of Laplace Transforms and the usual abstracts are allowed.

A• Determine the type of the equation (elliptic, parabolic or hyperbolic; Dirichlet homogeneous or non homogeneous) and solve the problem:

$$\begin{cases} 4\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial u}{\partial t} = 0; \quad 0 < x < 3 \ ; \ t > 0 \\ u(0,t) = 0; \quad u(3,t) = 0; \quad t > 0 \\ u(x,0) = 2x; \quad 0 < x < 3 \end{cases} \tag{1}.$$

B• Determine the type of the equation (elliptic, parabolic or hyperbolic; Dirichlet homogeneous or non homogeneous) and solve the problem :

$$\begin{cases} 4\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial^2 u}{\partial t^2} = 0; \quad 0 < x < 3 \ ; \ t > 0 \\ u(0,t) = 0; \quad u(3,t) = 0; \quad t > 0 \\ u(x,0) = -\sin(\pi x) + \dfrac{\pi}{2}\sin(2\pi x); \quad 0 < x < 3 \\ \dfrac{\partial u}{\partial t}(x,0) = 2\sin(\dfrac{2\pi x}{3}) + \pi \sin(\dfrac{5\pi x}{3}); \quad 0 < x < 3 \end{cases} \tag{2}.$$

C• Determine the type of the equation (elliptic, parabolic or hyperbolic;

Dirichlet homogeneous or non homogeneous) and solve the problem :

$$\begin{cases} 3\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial^2 u}{\partial t^2} = 0; \quad 0 < x < 2 \ ; \ t > 0 \\ \\ u(0,t) = 0; \quad u(2,t) = 0; \quad t > 0 \\ \\ u(x,0) = -\sin(\pi x) + \dfrac{\pi}{2}\sin(2\pi x) + \pi \sin(\dfrac{5\pi x}{2}); \quad 0 < x < 2 \\ \\ \dfrac{\partial u}{\partial t}(x,0) = 2\sin(\dfrac{3\pi x}{2}) + \pi \sin(\dfrac{5\pi x}{2}); \quad 0 < x < 2 \end{cases} \tag{2}.$$

D• Determine the type of the equation (elliptic, parabolic or hyperbolic; Dirichlet homogeneous or non homogeneous) and solve the problem:

$$\begin{cases} \pi^2 \dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial u}{\partial t} = 0; \quad 0 < x < 3 \ ; \ t > 0 \\ u(0,t) = 0; \quad u(3,t) = 0; \quad t > 0 \\ u(x,0) = x; \quad 0 < x < 3 \end{cases} \tag{1}.$$

E• Consider the problem (1) :

$$\begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{\partial^2 u}{\partial x^2} = 0; \quad 0 < x < 2 \ ; \ t > 0 \\ u(0,t) = 0; \quad u(2,t) = 0; \quad t > 0 \\ u(x,0) = -\sin(\dfrac{\pi x}{2}); \quad 0 < x < 2 \end{cases} \tag{1}.$$

1.a• Determine its type (elliptic, parabolic or hyperbolic);

1.b• Solve it.

F• Consider the problem (2):

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0 \quad 0 < x < 3;\ 0 < y < 5 \\ \begin{cases} u(x,0) = \pi; 0 < x < a \\ u(x,b) = 2\pi \sin(\pi x) \end{cases} \\ \begin{cases} u(0,y) = -2\sin(\pi x); 0 < y < b \\ u(a,y) = -1 \end{cases} \end{cases} \tag{2}.$$

2.a• Determine its type (elliptic, parabolic or hyperbolic);

2.b• Solve it.
