

MATH3224 - Final Exam Review Sheet - Summer 1- 04 - Konaté

Notice: Show your work. A right answer with a bad reasoning will be considered as wrong.

1 • determine the domain and the range of functions, $f(x) = \ln(|x|)$ and $g(x) = \frac{1}{x}$.

Determine the domain and range and find the functions $g \circ f$ and $f \circ g$.

2 • a and b are two positif real numbers such that $0 < a < b$. Montrer: $a < \sqrt{ab} < b$ and $\sqrt{ab} \leq \frac{1}{2}(a + b)$.

3 • Find the limit points of the sequence $a_n = \frac{(-1)^n}{1+n}$; $b_n = (-1)^n + \frac{1}{1+n}$; $c_n = \sin(\frac{n\pi}{4})$.

4 • Consider the function $f(x) = \frac{2}{5}x + 1$ on \mathbf{R} . Determiner δ such that $|x - 1| \leq \delta$ implies $|f(x) - f(1)| \leq \epsilon$ for ϵ given.

5 • Consider a function f continuous on the bounded interval $I = [a, b]$. Set

$$F(x) = \int_a^x f(t) dt$$

Prove F is Lipschitz continuous on I .

6 • Let $g_n(x) = \frac{x}{n} \exp(-\frac{x}{n})$. Prove that g_n converges pointwise.

7 • Determine if the series is convergent or not $\sum_{n=1}^{+\infty} \frac{2^{n+1}}{3^{2n}}$

8 • determine the open interval of the values of x for which the series $\sum_{n=1}^{+\infty} \frac{(3x - 2)^n}{n3^n}$ is absolutely convergent.

9 • Assume a and b are limit point to the sequences $\{a_n\}_{n \in \mathbf{N}}$ and $\{b_n\}_{n \in \mathbf{N}}$. Is $a + b$ limit point to $\{(a_n + b_n)\}_{n \in \mathbf{N}}$?
