MATH3224 - Review sheet for the Mid Term Exam - Summer 1- 04 - Konaté

Notice: Show your work. A right answer with a bad reasoning will be considered as wrong.

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1 • Show that a Cauchy sequence in $\mathbf{R}$ is bounded

2• Consider two Cauchy sequences of real numbers, say $\{a\}_{n\in\mathbf{N}}$ and $\{b\}_{n\in\mathbf{N}}$. Prove that the product of these 2 sequences is also a Cauchy sequence.

3 • Show that the sequence $\{a\}_{n\in\mathbf{N}}$ such that $a_n = (-1)^n - \frac{2}{n}$ has two limit points. (Find them and prove they are limit points).

4 • Prove

$$(\forall x \in \mathbf{R})(\forall y \in \mathbf{R})(0 < x < y \Longrightarrow x^2 < y^2).$$

5 • Prove $\{a\}_{n\in\mathbf{N}}$ such that $a_1 = \sqrt{2}$ and for $n \geq 2$, $a_{n+1} = \sqrt{2 + \sqrt{a_n}}$ is a Cauchy sequence which satisfies $\sqrt{2} \leq a_n \leq 2$ for all .

6 • Consider the function from $\mathbf{R}$ to $\mathbf{R}$ defined by $f(x) = x + 3$. Find the domain and the range of f. Prove that f is continuous. Is it uniformly continuous?

7 • Consider the function from [1,3] to $\mathbf{R}$ defined by $f(x) = x + 3$. Is it uniformly continuous? Find the domain and the range of f.

8 • Consider the function

$$f(x) = \begin{cases} 2 + x & \text{for} \quad x < 1 \\ \\ 5 - x & \text{for} \quad x > 1. \end{cases}$$

Prove that f is not continuous at the point $x = 1$.
