

MATH3224 - Graded Homework 4 - Summer 1- 04 - Konaté

Notice: Show your work. A right answer with a bad reasoning will be considered as wrong.

Responding right to each question is worth 25 points.

1 • Definition: A real valued function f defined on a set $S \subset \mathbf{R}$ is said to be Lipschitz continuous if:

$$(\exists M \in \mathbf{R})(M > 0)(\forall x \in S)(\forall \bar{x} \in S)(x \neq \bar{x}); \quad \frac{|f(x) - f(\bar{x})|}{|x - \bar{x}|} \leq M.$$

Show that if f is Lipschitz continuous on S then f is uniformly continuous on S .

2 •

Considering the first order Taylor sum

$$f(x+h) = f(x) + hf'(\xi); \quad \xi \in [0, h]$$

show that if a function f is continuously differentiable at a point \bar{x} then there is an interval I around \bar{x} such that f is Lipschitz continuous on I .

3 •

Do problem 8 of section 3.3 (page 94).

4 •

Do problem 9 of section 3.3 (page 94).
