MATH3224 - Graded Homework 1 - Summer 1- 04 - Konaté

Notice: Show your work. A right answer with a bad reasoning will be considered as wrong.

Responding right to each question is worth 20 points.

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1 • Consider the sets A and B such that

$$A = \{(x \in \mathbf{R}); \quad x^2 - 1 \geq 0\} \qquad B = \{(x \in \mathbf{R}); \quad x^2 - x - 6 = 0\}$$

Find $A \cup B$ and $A \cap B$.

2 • Consider
$A = \{-1, 2, 1, -2, 4, 3, 0\}$ and $B = \{(x \in \mathbf{R}); \quad 2x(1 - x^2) = 0\}$.
2.a Prove $B \subset A$ and $B \neq A$.
2.b Find the set C such that $C = A \setminus B$.

3 •
3.a Consider the function f from $\mathbf{R}$ to $\mathbf{R}$ such that

$$f(x) = \sqrt{2 - x^2}.$$

Determine $\mathcal{D}om(f)$ and $\mathcal{R}an(f)$.
3.b Write in terms of intervals of $\mathbf{R}$ the following set

$$A = \{(x \in \mathbf{R}); \quad |x - 2| \leq 1 \quad \text{and} \quad |x - \frac{1}{2}| < 2\}$$

4 •
4.a Prove that the sequence $\{a_n\}_{n \in \mathbf{R}}$ such that $a_n = \dfrac{1}{3 + \cos(n)}$ is bounded.

4.b Prove that the sequence $\{a_n\}_{n \in \mathbf{R}}$ such that $a_n = (3 - \frac{2}{n})(2 + \frac{5}{n})$ has a limit (to be found at first).

5 • Prove that the sequences $\{a_n\}_{n \in \mathbf{R}}$ and $\{b_n\}_{n \in \mathbf{R}}$ such that $a_n = \dfrac{\sin(1000n)}{n}$, $b_n = \dfrac{1}{n} + \dfrac{\cos(n)}{1 + n}$, have limits (to be found at first).
