PROSPECTUS IN SCIENTIFIC RESEARCH

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1 General Introduction

I am coming from the French school where a well-established tradition may be summarized by a word from the late and outstanding mathematician of French origin André Weil who have said: "A good mathematician does not need to publish too much". So most of my papers are still unpublished. I did and still do restrict myself to publishing only my papers that I do consider in line to bring a significant contribution to the fields under consideration. I must say that this tradition is changing rapidly based particularly on the outstanding and rich work output of J.L Lions laureate among others distinctions of the John Von Neumann Prize (1986) and the Prize of Japan and his son P.L. Lions a 1994 Fields Medal laureate.
The current prospectus describes my work as it has been over the past ten to fifteen years and sketches my contemplation for the years to come.

2 My early and past research works

Being an applied mathematician with a background in pure mathematics, my very early work in research was in optimization and Spline functions. Then followed a long period when I introduced myself to modeling in life sciences and did work on the development of certain kinds of Icterus in pregnant women and new born babies with the expectation to produce a diagnosis aided software that could help improving healthcare particularly in less equipped countries. At that time, primarily under the financial sponsorship of the French Ministry for Cooperation and Development, I discovered and worked on concrete problems of industrial concern as optimization of power production and power distribution. I worked also on the modeling of the Niger river, among other subject matters.

3 A spotlight on my current research work

These last fifteen years, my scientific work concerns the following academic fields :
- Asymptotic Analysis
- Numerical Analysis
- Operators and Abstract equations.

I specifically did pay attention to problems steming from engineering, biology, chemistry and nuclear physics and that are governed by some kind of ordinary or partial differential equations into which the highest derivative terms, at least, are affected by a very small parameter say ϵ or involve variables with very large and small scales. In other terms, my focus is on Singular Perturbation. The main objectives of my work may be introduced as follows.

1. I am giving a large interest to asymptotic analysis as a tool to reveal the particular behaviour of singularly perturbed problems over any particular suddomain of the geometric domain of study. Then comes my objective that is computation oriented and that is to use asymptotic analysis to work out easy to compute solutions to singularly perturbed problems. The solutions I am obtaining are to be compared to the usuall asymptotic solutions commonly present in the mathematical literature and that are often more symbolic and formel than computable.

2. Usually the asymptotic approach and the classical numerical approch are understood as being exclusive one to each other. So one objective of mine was to make use of my technique of precise localization of suddomains of "stiff behaviour" to make it possible to call on such classical numerical methods based on polynomial approximation as Collocation or Finite Element Method to compute efficiently the solution of a singularly perturbed equation. Then those classical methods are rendered competitive compared to special numerical techniques dedicated to singularly perturbed problems developped by some well-known authors like IL'In , Doedel , Russel and Shampine , Miller , Gartland Jr. , Kaddalbajoo and Reddy or Shiskin .
3. Besides these first objectives strongly computation oriented it appears to me the necessity to extend my work in asymptotics toward abstract differential equations in Banach and Hilbert spaces in order to involve the study of phenomena arising in mathematical physics, particularly that of particle collision in a dense and confined atmosphere arising in nuclear physics and transport theory to further some important results established by such authors as J.B Keller and E.W Larsen, JR Mika and J. Banasiak to describe the asymptotic behaviour of these collision phenomena in terms of the small parameter ϵ representing the scale of the colliding distance between two particles.

To illustrate those objectives and to establish my results, durant these past years, I did deal with a large variety of problems that I may summarize into the following list:

diffusion convection problem:

$$\begin{cases} L_\epsilon u \equiv \epsilon u'' + a(x)u' + b(x,u) = f \quad 0 \leq x \leq 1 \\ B_0 u = u(0) = A \\ B_1 u = u'(0) = \epsilon^{-1}B; \end{cases}$$

diffusion reaction problem

$$\begin{cases} L_\epsilon u \equiv \epsilon u^4 - (a(x)u'))' + b(x)u' + c(x)u = f \quad 0 \leq x \leq 1 \\ u(0) = u'(0) = u(1) = u'(1) = 0 \\ a(x) > \alpha > 0 \\ c(x) - \frac{1}{2}b'(x) \geq \beta > -\alpha; \end{cases}$$

a problem is Chemical Reactor Theory brought out by O'Malley and D.S Cohen:

$$\begin{cases} L_\epsilon u \equiv \epsilon u'' + a(x)u' + b(x,u) = f \quad 0 \leq x \leq 1 \\ B_{\epsilon,0}u = u(0) - \epsilon u'(0) = A \\ B_{\epsilon,1}u = u(1) + \epsilon u'(1) = B; \end{cases}$$

Stokes Problem that is a linear perturbed problem that appears naturally or at the semidiscretization of the classical Navier-Stokes problem sith small viscosity;

$$\begin{cases} -\epsilon\Delta u + u + gradp = f \quad over \quad \Omega \\ divu = 0 \quad over \quad \Omega \\ u = 0 \quad over \quad \Gamma; \end{cases}$$

Coupled equations to describe the two-scale physical phenomenon comprising a "slow" component that is the thermal absorption of neutrons and a fast component that is fission that happens in the core of a reactor in operation.

$$\begin{cases} L_{1,\epsilon}(x(t), y(t)) = \epsilon\partial_t x(t) - Ax(t) - Py(t) \\ L_{2,\epsilon}(x(t), y(t)) = \partial_t y(t) - Qx(t) - By(t)\,. \end{cases}$$

Consider $\mu_1 \in \mathcal{D}(A) \subset \mathbf{X}$; $\mu_2 \in \mathbf{X}$ where $\mathbf{X}$ is a given Banach space. Two "convenient" functions, say f and g being given, we consider the following coupled system of evolution equations :

$$\begin{cases} L_{1,\epsilon}(x(t),y(t)) \;=\; f(t) \\ L_{2,\epsilon}(x(t),y(t)) \;=\; r(t) \\ x(o) \;=\; \mu_1 \quad y(0) \;=\; \mu_2 \\ t \;\in]0,T[; \end{cases}$$

I also started these last five years or so to give an increasing interest to weather and climate modeling that report to equations with large scale variables and small scale ones.

3.1 My Personal Contributions:

My work consists in working on both numerical and asymptotic aspects of Singular Perturbation using my results in asymptotic analysis to make progress with numerical methods. The bridge that allows me to connect the two aspects is the precise localization of the boundary layers.

I have brought these last years
A. my contribution in Numerical Analysis, that consisted to:

- define a precise localization of the boundary layer. This have led to an efficient use of numerical methods based on polynomial approximation and a "natural" appearance of a Shiskin-type mesh . This method works even on problems in higher dimensions of space;
- define a notion of corrector which allows to calculate higher order solutions to both higher order ordinary differential equations and partial differential equations;

B. my contribution in Asymptotic Analysis, that consisted to:

- establish that in most situations, an asymptotic approximation to the solution of a perturbed equation may be obtained only on the basis of the outer expansion at the exclusion of the inner expansion which is the cause of the major numerical difficulties linked to the use of asymptotic techniques.

Eventually, my work have led to break through some unsolved problems.

1. I have solved the problem opened by J.L. Lions in 1973 that was to find a "corrector" to the perturbed Stokes problem defined on a bounded domain of $\mathbf{R}^2$ and $\mathbf{R}^3$, (cf. [1,5] below);

2. I have used the precise localization of the boundary layer to validate and use Collocation methods on both singularly perturbed problems of diffusion convection type and reaction diffusion type. These results in one dimension have been extended to higher dimensions in space (cf. [2] below)

3.I have shown that the corrector term in some singularly perturbed problems concerning particle collision in densely confined domains in nuclear physics and transport theory may be written only in terms of the outer expansion rather than two or more terms in addition to the outer expansion as it was stated previously (cf. [4,6] below)

4.I have constructed an efficient and easy to compute solution to the chemical reactor problem that was brought out by Robert O'Malley (cf. [3] below).

3.2 My Perspectives in Scientific Research

These results that led me to appear on the "Who's Who in Science and Technology in the World", 2002 edition, have also led the Department of Special Initiatives at Virginia Tech to designate me as leader of a new research group on the numerical analysis and the development of numerical methods on stiff equations arising in engineering and biology.

Within this interdisciplinary perspective, I envision , at first, to build a teaching manual that will allow the development of both an upper level undergraduate course and a graduate course in numerical analysis in connection with the departments of engineering and bioinformatics. The next points of my research agenda will be:

1. to further the use of the fruitful precise localization of the boundary layer in terms of computation capabilities for solving stiff differential equations describing phenomena that exhibit damping. In this order, I envision to pay a great deal of attention to problerms coming from biology since they are usually with damping and the quantity of chemicals involved in biological reactions varies sharply;
2. to stenghthen the combination between the use of asymtotic solutions and polynomial based numerical approximation schemes;
3. to pay more attention to multiple scale problems arising both in climate study and in biology that present a particular interest through their nonlinear aspects.

3.3 Main recent papers

A. Papers already published

[1]- Asymptotic Solution for the Perturbed Stokes Problem in 2D and 3D (Proceedings of the Royal Society of Edimburgh , 129A, 811-824, 1999)
[2]- Uniformly Convergent Schemes For Singularly Perturbed Differential Equations Based On Collocation Methods (International Journal of Mathematics and Mathematical Sciences , Vol 24, N5 (2000) Pp 305-313)
[3]-Strong Uniform Approximation for Some Singularly Perturbed Differential Equations Arising in Chemical Reactor Theory , to appear (Portugaliae Matematica)
[4]- Asymptotic Analysis Of Singularly Perturbed Abstract Evolution Equations in Banach and Hilbert Spaces, to appear (Turkish J. of Mathematics, Vol 26, 2002, Pp 1 - 20)

B. Preprints and Papers under review for publication)

[5]- Higher Order Asymptotic Approximation to the Stokes Problem, submitted
[6]- Asymptotic Analysis Of Singularly Perturbed Coupled Abstract Evolution Equations in Banach Spaces - Application to Transport Theory, submit-

ted

C. Main papers written for Conferences

[7]- A mathematical Model for the Climate Part 1 : A global Large Scale Model using a non linear Partial Differential Equation and its numerical analysis: a synthesis (Series of Conferences at Horizons mathmatiques , Universit de Bayonne 1994 France)

[8]- A Mathematical Model for the Climate : Part 2 : a local model using stochastic methods (Series of Conferences at Horizons mathmatiques , Universit de Pau , 1995 , France)

[9]- Strong Uniform Approximation To Singularly Perturbed Initial Value Problems (Invited Paper to the Tenth International Colloquium On Numerical Analysis and Applications, Plodiv Bulgaria, August 7-12, 2001) (non published)