

MATH4564 - Wave EQUATION - Konaté

Wave Equation with Homogeneous Boundary Conditions

Consider the following Wave Problem:

$$\left\{ \begin{array}{l} \alpha^2 u_{xx} = u_{tt} \quad 0 < x < L ; t > 0 \\ u(0, t) = 0 \quad u(L, t) = 0 \quad t > 0 \\ u(x, 0) = f(x) \\ u_t(x, 0) = g(x) \end{array} \right. \quad (1).$$

The solution $u(x, t)$ of equation (1) is:

$$\left\{ \begin{array}{l} u(x, t) = \sum_{m=1}^{+\infty} \left(a_m \cos \frac{m\pi at}{L} + b_m \sin \frac{m\pi at}{L} \right) \sin \frac{m\pi x}{L} \\ a_m = \frac{2}{L} \int_0^L (f(x)) \sin \frac{m\pi x}{L} dx \\ b_m = \frac{2}{m\pi a} \int_0^L (g(x)) \sin \frac{m\pi x}{L} dx \\ = 0, 1, 2, \dots \end{array} \right.$$

Remarks

If $f(x) = 0$ then $a_m = 0 \quad m = 1, 2, 3, \dots$

If $g(x) = 0$ then $b_m = 0 \quad m = 1, 2, 3, \dots$

Both $f(x)$ and $g(x)$ are considered being odd extensions over the interval $] -L, L[$.
