MATH4564 - LAPLACE EQUATION in 2D- Konaté

- **Dirichlet Non Homogeneous Boundary Conditions**

Consider the following (time independant) Laplace Problem in $\mathbf{R^2}$:

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0 \quad 0 < x < a;\ 0 < y < b \\ \begin{cases} u(x,0) = f_1(x); 0 < x < a \\ u(x,b) = f_2(x) \end{cases} \\ \begin{cases} u(0,y) = g_1(y); 0 < y < b \\ u(a,y) = g_2(y) \end{cases} \end{cases} \qquad (1).$$

The solution $u(x,y)$ of equation (1) is:

$$\begin{cases} u(x,y) = \displaystyle\sum_{m=1}^{+\infty} \left[\alpha_m \sinh(\frac{m\pi(b-y)}{a}) + \beta_m \sinh(\frac{m\pi y}{a}) \right] \sin(\frac{m\pi x}{a}) + \\ + \displaystyle\sum_{m=1}^{+\infty} \left[\gamma_m \sinh(\frac{m\pi(a-x)}{b}) + \delta_m \sinh(\frac{m\pi x}{b}) \right] \sin(\frac{m\pi y}{b}) \end{cases}$$

with

$\alpha_m = \frac{a_m}{A_1}$; $\beta_m = \frac{b_m}{A_1}$; $A_1 = \sinh(\frac{m\pi b}{a})$; $\gamma_m = \frac{c_m}{A_2}$; $\delta_m = \frac{d_m}{A_2}$; $A_2 = \sinh(\frac{m\pi a}{b})$

where:

$$\begin{cases} f_1(x) = \displaystyle\sum_{m=1}^{+\infty} a_m \sin(\frac{m\pi x}{a}); \quad a_m = \frac{2}{a}\int_0^L (f_1(x)) \sin\frac{m\pi x}{a} dx \\ f_2(x) = \displaystyle\sum_{m=1}^{+\infty} b_m \sin(\frac{m\pi x}{a}); \quad b_m = \frac{2}{a}\int_0^L (f_2(x)) \sin\frac{m\pi x}{a} dx \\ g_1(y) = \displaystyle\sum_{m=1}^{+\infty} c_m \sin(\frac{m\pi y}{b}); \quad c_m = \frac{2}{b}\int_0^L (g_1(y)) \sin\frac{m\pi x}{b} dy \\ g_2(y) = \displaystyle\sum_{m=1}^{+\infty} d_m \sin(\frac{m\pi y}{b}); \quad d_m = \frac{2}{b}\int_0^L (g_2(y)) \sin\frac{m\pi x}{b} dy \end{cases}$$

Observations: • f_1, f_2, g_1 and g_2 are considered as odd extensions.

• If any of those functions, say for example $f_1(x)$ is a finite sine series then the contribution of f_1, to the solution $u(x,y)$ also is a finite sine series.
