

MATH4564 - HEAT EQUATION No3- Konaté

Heat Equation with Newman Boundary Conditions

• 3- Newman Homogeneous Boundary Conditions

Consider the following Heat Problem:

$$\left\{ \begin{array}{l} \alpha^2 u_{xx} = u_t \quad 0 < x < L ; t > 0 \\ u_x(0, t) = 0 \quad u_x(L, t) = 0 \quad t > 0 \\ u(x, 0) = f(x) \end{array} \right. \quad (1).$$

The solution $u(x, t)$ of equation (1) is:

$$\left\{ \begin{array}{l} u(x, t) = \frac{c_0}{2} + \sum_{m=1}^{+\infty} c_m e^{\frac{-(m^2 \pi^2 \alpha^2)t}{L^2}} \cos \frac{m\pi x}{L} \\ c_m = \frac{2}{L} \int_0^L (f(x)) \cos \frac{m\pi x}{L} dx \\ m = 0, 1, 2, \dots \end{array} \right.$$

- f is considered as an even extension.
 - The quantity $\frac{c_0}{2} = \frac{1}{L} \int_0^L f(x) dx$ is called the Mean distribution of temperature.
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