

MATH4564 - HEAT EQUATION No2- Konaté

Heat Equation with Dirichlet Boundary Conditions

- 2- Dirichlet Non Homogeneous Boundary Conditions

Consider the following Heat Problem:

$$\begin{cases} \alpha^2 u_{xx} = u_t & 0 < x < L ; t > 0 \\ u(0, t) = T_1 & u(L, t) = T_2 & t > 0 \\ u(x, 0) = f(x) \end{cases} \quad (1).$$

The solution $u(x, t)$ of equation (1) is:

$$\begin{cases} u(x, t) = \frac{(T_2 - T_1)x}{L} + T_1 + \sum_{m=1}^{+\infty} c_m e^{\frac{-(m^2 \pi^2 \alpha^2)t}{L^2}} \sin \frac{m\pi x}{L} \\ c_m = \frac{2}{L} \int_0^L \left(f(x) - \frac{(T_2 - T_1)x}{L} - T_1 \right) \sin \frac{m\pi x}{L} dx \\ m = 1, 2, 3, \dots \end{cases}$$
