MATH4564 - HEAT EQUATION No1- Konaté

Heat Equation with Dirichlet Boundary Conditions

- **1- Dirichlet Homogeneous Boundary Conditions**

Consider the following Heat Problem:

$$\begin{cases} \alpha^2 u_{xx} = u_t \quad 0 < x < L \ ; \ t > 0 \\ u(0,t) = 0 \quad u(L,t) = 0 \quad t > 0 \\ u(x,0) = f(x) \end{cases} \qquad (1).$$

The solution $u(x,t)$ of equation (1) is:

$$\begin{cases} u(x,t) = \sum_{m=1}^{+\infty} c_m e^{\dfrac{-(m^2\pi^2\alpha^2)t}{L^2}} \sin \dfrac{m\pi x}{L} \\ c_m = \dfrac{2}{L}\int_0^L (f(x)) \sin \dfrac{m\pi x}{L} dx \\ m = 1,\ 2,\ 3, \cdots \end{cases}$$

Observations:

- f **is considered as an odd extension.**
- If function $f(x)$ is a finite sine series then the solution $u(x,t)$ also is a finite sine series. More precisely, if: $f(x) = \sum_{m=1}^{q} b_m \sin \dfrac{m\pi x}{L}$ then:

$$u(x,t) = \sum_{m=1}^{q} b_m e^{\dfrac{-(m^2\pi^2\alpha^2)t}{L^2}} \sin \dfrac{m\pi x}{L}.$$
