FINITE DIFFERENCE

Consider the second order differential equation:

$$\text{(1)} \begin{cases} a(t)y''(t)+b(t)y'(t)+c(t)y(t)=f(t) \\ y(0)=y_0 \qquad y'(0)=y_0' \end{cases}$$

A Numerical solution (approximation solution) to (1) may be built over a finite interval $I=[0,T]$ through the EXPLICIT FINITE DIFFERENCE SCHEME:

(1) discretize I with a mesh size $h=\frac{T}{N}$; we set the following discrete approximation to the derivatives of $y(t)$:

$$y'(t_i)=y_i'=\frac{y_{i+1}-y_i}{h} \quad \text{where } y(t_i)=y_i;\ t_i=\text{mesh point}$$

$$y''(t_i)=y_i''=\frac{y_{i+1}-2y_i+y_{i-1}}{h^2}\ ;\ \text{so (1) is replaced}$$

with the following discrete model:

$$\begin{cases} y(0)=y_0 \\ y_1=hy_0'+y_0 \\ \text{For } i=1 \text{ to } N-1 \\ y_{i+1}=\dfrac{1}{\left[\frac{a_i}{h^2}+\frac{b_i}{h}\right]}\left[f_i-y_i\left(c_i-\frac{2a_i}{h^2}-\frac{b_i}{h}\right)-\frac{a_i}{h^2}y_{i-1}\right] \end{cases}$$