

MATH4564 - Laplace Transform- An Abstract No2- Konaté

For a second order ordinary differential equation, the work starts with the formulas: $\mathcal{L}\{y''(t)\}(s) = s^2Y(s) - sy(0) - y'(0)$; $\mathcal{L}\{y'(t)\}(s) = sY(s) - y(0)$

Procedure

1– Take the Laplace Transform (L.T.) of the given equation

2– Construct $Y(s)$ in the form of a **rational function** (a quotient of 2 polynomials):

$$Y(s) = \frac{P(s)}{Q(s)}, \quad \text{where } d^\circ P > d^\circ Q$$

3– Find $y(t)$ in taking the Inverse Laplace Transform (I.L.T.) of $Y(s)$ which you express as a sum of **partial fractions**. This step depends on the form of $Q(s)$.

3.1– $Q(s)$ has only single real roots. For example, assume $Q(s)$ has only three single roots a_1 , a_2 and a_3 then:

$$Y(s) = \frac{P(s)}{(s - a_1)(s - a_2)(s - a_3)} \text{ and determine the coefficients } A_1, A_2 \text{ and } A_3 \text{ such that: } Y(s) = \frac{A_1}{s - a_1} + \frac{A_2}{s - a_2} + \frac{A_3}{s - a_3}$$

3.2– $Q(s)$ contains an **irreducible quadratic factor** of the form $(s^2 + a^2)$ or $((s - a)^2 + b^2)$.

Assume $Q(x) = (s - a_1)(s^2 + a^2)((s - a_2)^2 + b_2^2)$ then its expression as partial fractions will be:

$$Q(s) = \frac{A_1}{s - a_1} + \frac{As + B}{s^2 + a^2} + \frac{A_2(s - a_2) + B_2}{(s - a_2)^2 + b_2^2}.$$

3.3– $Q(s)$ contains repeated factors.

Assume $Q(x) = (s - a_1)^3(s^2 + a^2)^3((s - a_2)^2 + b_2^2)^3$. Then it may be expressed as follows:

$$Q(s) = \frac{A_{11}}{s - a_1} + \frac{A_{12}}{(s - a_1)^2} + \frac{A_{13}}{(s - a_1)^3} + \frac{A_1s + B_1}{(s^2 + a^2)} + \frac{A_2s + B_2}{(s^2 + a^2)^2} + \frac{A_3s + B_3}{(s^2 + a^2)^3} + \frac{A_{21}(s - a_2) + B_{21}}{(s - a_2)^2 + b_2^2} + \frac{A_{22}(s - a_2) + B_{22}}{((s - a_2)^2 + b_2^2)^2} + \frac{A_{23}(s - a_2) + B_{23}}{((s - a_2)^2 + b_2^2)^3}.$$