Math2214- Abstract For 2nd order Diff.Eq - Konaté

2. Non Homogeneous equations:

2.1 Homogeneous equations has constant coefficients:

Standard form: $ay'' + by' + cy = g(t)$; a, b, c are constants, $g(t)$ is regular function.General solution $y(t)$ is such that:

$y(t) = y_h(t) + y_p(t)$ where $y_h(t)$ = the general solution of the homogeneous equation and $y_p(t)$ = particular solution of non homogeneous equation. To find $y_p(t)$, apply the **Method of Undetermined coefficients**.

Method of Undetermined coefficients 2.1.1 $g(t) = P_n(t)$. Assume $P_n(t) = a_0 + a_1t + a_2t^2 + \ldots + a_nt^n$

If $r = 0$ is a root of multiplicity s of the characteristic equation ($s = 0$ or $s = 1$ or $s = 2$ then $y_p(t) = t^s(A_0 + A_1t + A_2t^2 + \ldots + A_nt^n)$. **2.1.2** $g(t) = P_n(t)\mathbf{e}^{\alpha}$. Assume $P_n(t) = a_0 + a_1t + a_2t^2 + \ldots + a_nt^n$

If $r = \alpha$ is a root of multiplicity s of the characteristic equation ($s = 0$ or $s = 1$ or $s = 2$ then $y_p(t) = t^s(A_0 + A_1t + A_2t^2 + \ldots + A_nt^n)e^{\alpha}$. **2.1.3** $g(t) = P_n(t)\mathbf{e}^{\alpha t}\cos(\beta)$ **or** $g(t) = P_n(t)\mathbf{e}^{\alpha t}\sin(\beta)$. Assume $P_n(t) = a_0 + a_1t + a_2t^2 + \ldots + a_nt^n$

If $r = \alpha + i\beta$ is a root of multiplicity s of the characteristic equation ($s = 0$ or $s = 1$ then

$y_p(t) = t^s(A_0 + A_1t + A_2t^2 + \ldots + A_nt^n)e^{\alpha t}\cos(\beta) + t^s(B_0 + B_1t + B_2t^2 + \ldots + B_nt^n)e^{\alpha t}\sin(\beta)$.

2.2 Method of Variation of Parameters: This method is universal but is applied mainly to non homogeneous equations with variable coefficients to which the method of undermined coefficients does not work.

Standard form of the non homogeneous equation with variable coefficients: $y'' + p(t)y' + q(t)y = g(t)$; $p(t), q(t), g(t)$ are regular functions.

Assume we know 2 solutions $y_1(t)$ and $y_2(t)$ of the homogeneous equation. Then general solution of the non homogeneous one is: $y(t) = u_1(t)y_1(t) + u_2(t)y_2(t)$. The u_1' and u_2' are solution of:

$$\begin{cases} u_1'y_1 + u_2'y_2 = 0 \\ u_1'y_1' + u_2'y_2' = g(t). \end{cases}$$
