Math2214- Abstract For 2nd order Diff.Eq - Homogeneous- Konaté

0. Pre-requisite: For 2 functions $y_1(t)$ and $y_2(t)$ the Wronskian is defined as being the following determinant:

$$W(y_1, y_2)(t) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

The following proposition is true:
$y_1(t)$ and $y_2(t)$ are linearly independent $\Leftrightarrow$ $W(y_1, y_2)(t) \neq 0$ for all t.

1. Homogeneous equations:
Definition: 2 solutions of an homogeneous equations which are linearly independent are said to constitute a set of fundamental solutions.
1.1 Homogeneous equations with constant coefficients:
Standard form: $ay'' + by' + cy = 0$; $\quad a$, b, c are constants.
Characteristic equation (c.e): $ar^2+br+c = 0$ with discriminant $\Delta = b^2-4ac$.
1.1.1: $\Delta > 0$ Then (c.e) has 2 singles roots, $r_1 = \dfrac{-b+\sqrt{\Delta}}{2a}$ and $r_2 = \dfrac{-b-\sqrt{\Delta}}{2a}$; 2 fundamental solutions are $y_1(t) = \mathrm{e}^{r_1 t}$ and $y_2(t) = \mathrm{e}^{r_2 t}$. General solution is $y_h(t) = C_1 y_1(t) + C_2 y_2(t)$.

1.1.2: $\Delta = 0$ Then (c.e) has 1 double roots, $r = -\dfrac{b}{2a}$; 2 fundamental solutions are $y_1(t) = \mathrm{e}^{rt}$ and $y_2(t) = t\mathrm{e}^{rt}$. General solution is $y_h(t) = C_1 y_1(t) + C_2 y_2(t)$.

1.1.3: $\Delta < 0$ Then the 2 roots of the (c.e) are complex and conjugate of each other. Set $r = \dfrac{-b+\sqrt{\Delta}}{2a} = \alpha + i\beta$ 2 fundamental solutions are $y_1(t) = \mathrm{e}^{\alpha t}\cos(\beta t)$ and $y_2(t) = \mathrm{e}^{\alpha t}\sin(\beta t)$. General solution is $y_h(t) = C_1 y_1(t) + C_2 y_2(t)$.

1.1 Homogeneous equations with variable coefficients: Standard form: $y'' + p_1(t)y' + p_2(t)y = 0$; $\quad p_1$, p_2 are regular functions.
Assume we know one solution $y_1(t)$. Then apply method of reduction of order to find $y_2(t) = v(t)y_1(t)$ where:

$$\begin{cases} u = v' \\ y_1 u' + (2y_1' + p_1 y_1)u = 0 \end{cases} \bullet$$