MATH2214 - Systems of First ODE - Abstract - Konaté

Solution of $X' = AX$; (1) A; 2×2 **real matrice**

0. Reminder:

λ is an **eigenvalue** to a square matrice A associated with the **eigenvector** X if $AX = \lambda X$. The complex numbers λ are the complex solutions of $\det(A_\lambda) = 0$ where $A_\lambda = A - \lambda I$.
We have $AY = \lambda Y$; $Y = kX$; $k \in \mathbf{R}$. If A is real then $A\overline{X} = \overline{\lambda X}$.

1. A has two 1-fold real eigenvalues

λ_1 and λ_2, are the 2 eigenvalues associated to the eigenvectors $\vec{u}_1$ and $\vec{u}_2$.
If $\det[\vec{u}_1(t), \vec{u}_2(t)] \neq 0$ then the general solution of (1) is:
$X(t) = C_1\vec{u}_1 \mathrm{e}^{\lambda_1 t} + C_2\vec{u}_2 \mathrm{e}^{\lambda_2 t}$

2. A has two 1-fold complex (conjugate) eigenvalues

λ is one eigenvalue of A associated with the eigenvector $\vec{u}$. Call $Y_1(t)$ and $Y_2(t)$ the real part and the complex part of $Y(t) = \vec{u}\mathrm{e}^{\lambda t} = Y_1(t) + iY_2(t)$.
If $\det[Y_1(t), Y_2(t)] \neq 0$ then the general solution of (1) is
$X(t) = C_1Y_1(t) + C_2Y_2(t)$.

3. A has one 2-fold real eigenvalue

λ is the eigenvalue of A associated with the eigenvector $\vec{u}$. $\vec{v}$. such that $A_\lambda\vec{v} = \vec{u}$ is called a **generalized eigenvalue of** A.. Set $\vec{v} = t\vec{u} + \vec{v}_0$.
If $\det[\vec{u}(t), \vec{v}(t)] \neq 0$ then the general solution of (1) is:
$X(t) = C_1\vec{u}\mathrm{e}^{\lambda t} + C_2\vec{v}_2\mathrm{e}^{\lambda t}$
