Math2214- Abstract- Higher ODE - Konaté

2. Non Homogeneous equations:

2.1 Homogeneous equations has constant coefficients:
Standard form: $a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y = 0. = g(t)$ $y(t)$ is such that:
$y(t) = y_h(t) + y_p(t)$ where $y_h(t)$ = the general solution of the homogeneous equation and $y_p(t)$ = particular solution of non homogeneous equation. To find $y_p(t)$, apply the **Method of Undetermined coefficients**.
Method of Undetermined coefficients
2.1.1 $g(t) = P_n(t)$. Assume $P_n(t) = c_0 + c_1 t + c_2 t^2 + \ldots + c_n t^n$
If $r = 0$ is a s-fold root of the characteristic equation ($s = 0, 1, ..., n$) then $y_p(t) = t^s(A_0 + A_1 t + A_2 t^2 + \ldots + A_n t^n)$.

2.1.2 $g(t) = P_n(t)\mathbf{e}^{\alpha}$. Assume $P_n(t) = c_0 + c_1 t + c_2 t^2 + \ldots + c_n t^n$
If $r = \alpha$ is a s-fold root of the characteristic equation ($s = 0, 1, ..., n$) then $y_p(t) = t^s(A_0 + A_1 t + A_2 t^2 + \ldots + A_n t^n)\mathrm{e}^{\alpha}$.

2.1.3 $g(t) = P_n(t)\mathbf{e}^{\alpha t}\cos(\beta)$ **or** $g(t) = P_n(t)\mathbf{e}^{\alpha t}\sin(\beta)$. Assume $P_n(t) = c_0 + c_1 t + c_2 t^2 + \ldots + c_n t^n$
If $r = \alpha + i\beta$ is a s-fold root of the characteristic equation ($2s \leq n$) then $y_p(t) = t^s(A_0 + A_1 t + A_2 t^2 + \ldots + A_n t^n)\mathrm{e}^{\alpha t}\cos(\beta) + t^s(B_0 + B_1 t + B_2 t^2 + \ldots + B_n t^n)\mathrm{e}^{\alpha t}\sin(\beta)$.

2.2 Method of Variation of Parameters: This method is universal but is applied mainly to non homogeneous equations with variable coefficients to which the method of undermined coefficients does not work.
