

Math2214- Abstract - Higher ODE - Homogeneous- Konaté
Homogeneous Equations

0. Pre-requisite: For n functions $y_1(t), y_2(t) \cdots y_n(t)$ the Wronskian is defined as being the following determinant:

$$W(y_1, y_2, \dots, y_n)(t) = \begin{vmatrix} y_1 & y_2 & \cdots & y_n \\ y_1' & y_2' & \cdots & y_n' \\ \dots & \dots & \dots & \dots \\ y_1^{(n)} & y_2^{(n)} & \cdots & y_n^{(n)} \end{vmatrix}$$

The following proposition is true:

$y_1(t), y_2(t) \cdots y_n(t)$ are linearly independent $\Leftrightarrow W(y_1, y_2, \dots, y_n)(t) \neq 0$ for all t .

1. Homogeneous equations:

Definition: n solutions of an homogeneous equations which are linearly independent are said to constitute a set of fundamental solutions.

1.1 Homogeneous equations with constant coefficients:

Standard form: $a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y = 0$.

Characteristic equation (c.e): $Z(r) = a_n r^n + a_{n-1} r^{n-1} + \cdots + a_1 r + a_0 = 0$.

1.1.1: $Z(r) = 0$ has single roots Then the contribution of the root r_i to $y_h(t)$ will be $y_i(t) = e^{r_i t}$ and General solution is $y_h(t) = \sum_{i=1}^n C_i y_i(t)$.

1.1.2: Multiple roots assume r_k is a s -fold root of $Z(r) = 0$. the contributions of r_k will be s functions $y_{k,i}$ $i = 0, 1, \dots, s-1$ where $y_{k,i} = t^s e^{r_k t}$. Then General solution is linear combination of all contributions

1.1.3: Complex roots Assume $r_k = \alpha + i\beta$ is a s -fold root of $Z(r) = 0$ $2s \leq n$ the contributions of r_k will be $2s$ functions $y_{k,1,i}; y_{k,2,i}$ $i = 0, 1, \dots, s-1$ where $y_{k,1,i} = t^s e^{\alpha t} \cos(\beta t)$; $y_{k,2,i} = t^s e^{\alpha t} \sin(\beta t)$.