

MATH2214 - Abstract- Linear First ODE- Konaté

A. Linear First Order Differential Equations

- **A.1 Case of constant coefficients**

Solve:

$$\frac{dy}{dt} = ay - b$$

where $a, b \in \mathbf{R}$.

The general solution is:

$$y(t) = ke^{at} + \frac{b}{a},$$

k is a constant. If $y(0) = y_0$ then:

$$y(t) = \left(y_0 - \frac{b}{a}\right)e^{at} + \frac{b}{a}.$$

- **A.2 Case of variable coefficients**

Solve:

$$\frac{dy}{dt} + p(t)y = g(t).$$

Use integrating factor or Leibniz method to find the general solution:

$$\begin{cases} y(t) = \frac{c + \int_{t_0}^t \mu(s)g(s)ds}{\mu(t)} \\ \mu(t) = e^{\int_{t_0}^t p(s)ds} \end{cases}$$

where c is a constant.

B. Non Linear First Order Differential Equations

- **Case of separated variables**

Solve:

$$M(t) + N(y)\frac{dy}{dt} = 0$$

The general (implicit) solution is:

$$H(y) + G(t) = c, \quad H(y) = \int N(u)du, \quad G(t) = \int M(u)du$$

c is a constant.