MATH2214 - Fall 03 Review For Final Exam- Konaté

Notice: Show your work. A right answer with a bad reasoning will be considered as wrong. No notebook allowed. Abstracts are allowed.

1• Solve

$$\begin{cases} t\dfrac{\mathrm{d}y}{\mathrm{d}t} - y = \dfrac{t^3}{t^2+1} \\ \\ y(1) = 0 \end{cases}$$

2• Find an implicite solution to

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\cos x}{5y^4}.$$

3• Determine the order of the equation and if it is linear or not, homogeneous or not:

$y^{(3)} + ty' + (\cos^2 t)y = t^3$

$(1+x^2)y'' + \mathrm{e}^{-x}y' + y = \mathrm{e}^x$

4• the decay constant for the Thorium 234 is $0.02828\,\mathrm{days}^{-1}$. This Thorium is replenished at a rate of $2\,\mathrm{mg}$ per day. Find the equation describing the quantity of Thorium.

5• $Q(t)$ A tank initially contains 200 liters of pure water. A mixture containing a concentration of 2 g/liter of salt enters the tank at a rate of 2 liters/min and the well-stirred mixture leaves the tank at the same rate. $Q(t)$ denotes the amount of salt in the tank at the time t. Find the equation describing the evolution of $Q(t)$.

6• Say if the equation $\mathrm{e}^t y'' + \sin(t)y = 1$ is: linear or non-linear; homogeneous or non-homogeneous; separable or not separable.

7• Solve the differential equation:

$y^{(4)} - y = 0; \qquad y'' + 5y + 4y = 0.$

8• Find a particular solution to $y'' + 25y = 6\sin t$

9• Write in the form of a system of linear first order differential equations:

$y^{(3)} + 10y'' - 9y' + 3y = \cos t$

10• Consider the differential equation $ty''+2(1-t)y'+(t-2)y=0; \quad t>0.$
Assuming that $y(t)=\mathrm{e}^t$ is one of its fundamental solution; write (only) the equation satisfied by the second when using the method of reduction of order.

11• Consider the differential equation $t^2y''-2y=3t^2-1; \quad t>0.$
Assuming that two fundamental solutions of its associated homogeneous equation are $y_1(t)=t^2$ and $y_2(t)=t^{-1}$ find the system of equations obtained by the method of variation of parameter and leading to its solution.

12• A system of 2 linear differential equations with real coefficients has a complex eigenvalue $\lambda=2+3\mathrm{i}$ and an associated eigenvector $\vec{u}=\begin{pmatrix}\mathrm{i}\\ \\ 1\end{pmatrix}$.
Find the general solution of the differential system $X'=AX$

13• Solve

$$X'=\begin{pmatrix}1 & 1\\ \\ 4 & 1\end{pmatrix}X.$$

14• Without calculating the coefficients, give the expression of the general solution of
$y''+3y'+2y=\mathrm{e}^{-t}+\mathrm{e}^{-2t}.$

15• $y_1(t)=\mathrm{e}^{2t}$ and $y_2(t)=\mathrm{e}^{-3t}$ form a set of fundamental solutions for the differential equation
$y''+a_1y'+a_0y=0$ where a_0 and a_1 are constants. Find a_0 and a_1.

16• Without solving the problem, find an interval in which the solution of the initial value problem is certain to exist:
$(4-t^2)y'+2ty=3t^2, \quad y(1)=-3$

17• Use the Euler Method, taking one step to calculate an approximation to $y(2.3)$ knowing that

$$\begin{cases}\dfrac{\mathrm{d}y}{\mathrm{d}t}=t^2y^3\\ \\ y(2)=-1.\end{cases}$$

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