MATH 5114 FINAL EXAM SPRING 2008

NAME ____(please print)

Four-hour time limit, closed book, closed notes. Hand test in flat, with the test sheet on top, and stapled in the upper left corner, to 568 McBryde Hall by noon on Tuesday, May 6, 2008. Show all work to receive full credit. Giving or receiving assistance on this test (except from the instructor) is a violation of honor system policies.

WORK ANY SIX (6) OF THE FOLLOWING SEVEN (7) PROBLEMS.

1. Let R be a unique factorization domain and let d be a nonzero element of R. Prove that there are only finitely many distinct principal ideals containing (d).

2. Let E/F be an extension of fields such that $[E : F] = 2$. Prove that there exists $\alpha \in E - F$ such that $\alpha^2 \in F$.

3. Let R be a ring with 1. A left R-module M is *simple* if its only submodules are 0 and itself. Prove that if M is simple, then M is cyclic, and every R-module homomorphism of M to itself is either the zero map or an isomorphism.

4. Let F be a field and let V be a vector space over F. Suppose that $\{a, b, c\}$ is a linearly independent subset of V. Characterize, with proof, the fields F for which $\{a+b, a+c, b+c\}$ is a linearly independent subset of V.

5. Let F/K be an extension of fields such that F is algebraic over K. Let D be an integral domain such that $K \subseteq D \subseteq F$. Prove that D is a field.

6. Let F be a field. Prove that the set R of polynomials in $F[x]$ whose coefficient of x is equal to 0 is a subring of $F[x]$ and that R is not a unique factorization domain.

7. Determine the splitting field, and its degree over $\mathbb{Q}$, for the polynomial $x^4 + x^2 + 1$.

On my honor I have neither received nor given assistance on this test.

Signature: ____