As you have learned, the Riemann integral of a function $f(x)$ over an interval $[a, b]$ is defined to be a certain limit of a Riemann sum of $f(x)$ as the lengths of the subintervals in the partition of $[a,b]$ go to zero. For most functions, computing this limit directly (and thereby computing the Riemann integral of $f$) is difficult, if not impossible. In fact, though Greek mathematicians more than 2,000 years ago knew that the answer to certain types of problems could, in theory, be found by computing the limit of such sums, the actual numerical computation of the limits involved were, in most cases, beyond their abilities. It was not until the eighteenth century that a way to (indirectly) overcome this difficulty was discovered by Isaac Newton, who proved what we now call the Fundamental Theorem of Calculus. Namely, the integral of the function $f(x)$ over the interval $[a,b]$ is the number $F(b) - F(a)$, where $F(x)$ can be taken to be any function for which $F'(x) = f(x)$ for all $x$ in $(a,b)$, a so-called "antiderivative" of the function $f(x)$.

In order to use this result to compute the value of an integral we need to be able to find such an antiderivative $F(x)$ of the integrand $f(x)$. For many common functions this turns out to be quite elementary, while for others the use of simple techniques such as partial fractions or substitution is often effective in transforming the problem into an equivalent one for which an antiderivative is easily found. However, it is also true that in many cases finding an antiderivative for a given function $f(x)$ can be a very difficult and/or time-consuming exercise.

In such cases Matlab (or other such mathematical software) can be of great value, having a built-in program for evaluating the definite integral of a function $f(x)$ on $[a,b]$ (using the Fundamental Theorem of Calculus, just as we would do it by hand). Therefore, as long as the function $f(x)$ has an antiderivative that is contained in the vast collection of such functions in the program's memory, the command `int(f,a,b)` will give the exact value of the Riemann integral of $f(x)$ on the interval $[a,b]$ where $f$ should be a symbolic expression for the function to be integrated.

**Example 1:** Find the integral of the function $f(x) = \cos x$ over the interval $\left[\frac{\pi}{3}, \frac{\pi}{2}\right]$.

```
>> sym x
>> int(cos(x), pi/3, pi/2)
ans = 1 - 1/2 * 3^(1/2)
```

Using Matlab to evaluate integrals this way is often a convenient time-saver even when, as in the above example, we could find an antiderivative ourselves rather easily.
**Example 2:** Find the integral of the function \( f(x) = \frac{x}{x^2 + 1} \) over the interval \( 0, \frac{2}{\sqrt{\pi}} \). (Note that in this example the antiderivative is not simple to find, as in example 1.)

\[
\int \frac{x}{x^2 + 1} \, dx \bigg|_{0}^{2/\sqrt{\pi}}
\]

\[
\text{ans} = 1/2 \cdot \text{atan} (25824365969885543609319430368361/20282409603651670423947251286016)
\]

This number is the EXACT value of the integral. In cases where we want a numerical value, the command `double(ans)` will give a decimal approximation to the exact value of the integral. The command means compute a numerical approximation to the expression obtained by the immediately preceding command.

In the example above this value is:

\[
\text{» double(ans)}
\]

\[
\text{ans} = 0.4525
\]

The default answer in Matlab is only four decimal places. If you want to show more decimal places, you can type `format long` into Matlab.

\[
\text{» format long}
\]

\[
\text{» double(ans)}
\]

\[
\text{ans} = 0.45251128838327
\]

Of course, the real value of Matlab lies in its ability to compute the value of integrals which we either do not see how to do by hand, or which would involve a tedious and time-consuming calculation if we could. A good example of this is the following integral which one could evaluate by repeated applications of integration by parts, but the calculations are sufficiently tedious that no one would want to attempt it.

**Example 3:** Find the integral of the function \( f(x) = x^{10} \cos x \) over the interval \([1,4]\).

\[
\int x^{10} \cos x \, dx \bigg|_{1}^{4}
\]

\[
\text{ans} = 2488576*\sin(4)-2401280*\cos(4)+1960649*\sin(1)-3053530*\cos(1)
\]