

RECITATION 8

Suppose a projectile is fired from ground level with a speed v_0 at an angle θ as measured from the horizontal.

- (1) Find a formula for the trajectory $(x(t), y(t))$.
- (2) Set the height $y(t)$ equal to zero and find the time that the projectile hits the ground.
- (3) Put this time into the formula for $x(t)$ to find a formula for the range, which will be in terms of θ and v_0 .
- (4) To find the maximum for a continuous function (if it has one), you can set its derivative equal to zero. Using the result obtained in (3), take the derivative of the range with respect to theta and set it equal to zero. Solve to find what angle θ will give maximum range.

Now suppose the projectile is fired from 1600 feet above ground level, with a speed of $32\sqrt{2}$ feet per second.

- (5) If the projectile is fired at an angle of 45 degrees, compute the range.
- (6) If the projectile is fired at an angle of 30 degrees (from horizontal), compute the range.
- (7) Can you tell from this if, in computing the angle for maximum range in part (4), the assumption that the projectile was fired from ground level is important?

HINTS FOR INSTRUCTORS

It is very important that we do not let them just pull out formulas memorized in high school, like

$$x = x_0 + v_0 t + \frac{1}{2} g t^2$$

We should insist that they begin by writing down initial conditions (in vector form) and the acceleration vector \vec{a} , and then integrate \vec{a} to get the vectors \vec{v} and \vec{r} .