RECITATION 7

It is easy to tell if a point $P = (p_1, p_2, p_3)$ is in a plane: just substitute for x, y, z and see if the equation of the plane is satisfied. It is more difficult to tell if a vector $\vec{v} =< v_1, v_2, v_3 >$ is in the plane, since the *name* of the vector refers to a vector describable as having its tail at the origin and head at the *point* $P = (v_1, v_2, v_3)$. In fact, the point $P = (\alpha, \beta, \gamma)$ and the vector $\vec{v} =< \alpha, \beta, \gamma >$ will not be in the same plane unless the plane passes through the origin.

1. Consider the plane $2x - y + 3z = 7$.

(a) Show that the point $P = (1, 1, 2)$ and the point $Q = (2, 0, 1)$ both lie in the plane.

(b) Show that the vectors $\vec{u} =< 1, 1, 2 >$ and $\vec{v} =< 2, 0, 1 >$ (same coordinates as points P and Q from part (a)) do not lie in the plane, but the vector $\vec{w} =< 1, -1, -1 >$ does lie in the plane. (Hint: test the vectors with the normal vector to the plane) Note that $\vec{w}$ is just $Q - P$. Why should the difference of two points in a plane lie in the plane, while the points, written as vectors, do not?

2. Unless two planes are parallel (or coincide), they will intersect in a line. Consider the two planes determined by $ax + by + cz = d$ and $\hat{a}x + \hat{b}y + \hat{c}z = \hat{d}$. To find their line of intersection, replace one variable, for example y, with t. Then you will have two simultaneous equations for x and z. Solving them will give you both x and z in terms of t (and you already have $y = t$). Thus you will have the parametric equations for a line.

(a) Find the parametric equations for the line of intersection of the plane $x + y + 2z = 2$ and the plane $x - y + 3z = 3$.

(b) Show that the point corresponding to $t = 1$ in your parametric equations is a point lying in both given planes (as it must).

3. You already know how to find the vector projection of a vector onto another vector. The vector projection of a vector $\vec{v}$ onto a plane is the vector formed by the shadow of $\vec{v}$ in the plane if a light is shined on $\vec{v}$ perpendicular to the plane. To find the vector projection of a vector $\vec{v}$ onto a plane, it is sufficient to find the vector projections of $\vec{v}$ onto any two vectors lying in the plane which are perpendicular to each other, and then adding these vector projections. However, this would require you first to find the two perpendicular vectors in the plane. An easier method is to find the vector projection of $\vec{v}$ onto a vector perpendicular to the plane, and then subtract the result from v. The difference will be the vector projection of $\vec{v}$ onto the plane.

(a) Find the vector projection of the vector $\vec{v} =< 2, 5, 4 >$ onto the plane $x - 2y + 3z = 2$ by the method suggested above. I.e., find the vector projection of $\vec{v}$ onto the normal to the given plane, and subtract this vector from $\vec{v}$ itself.

I did some work at home and found that the vectors $\vec{q} =< -1, 1, 1 >$ and $\vec{p} =< 5, 4, 1 >$ lie in the plane $x - 2y + 3z = 2$.

(b) Find the vector projection of $\vec{v}$ onto $\vec{q}$ and onto $\vec{p}$.

(c) Add the two vectors found it part (b). Do you get the same vector found in part (a)?

HINTS FOR INSTRUCTORS

Teaching about vectors in R^n in class is made a little tricky by the fact that the vast majority of students have already taken MATH 1114, but there are usually a few who have not.

I would draw the graph of a plane, with two points P and Q in it. Then I would draw vectors from the origin to P and Q and sketch in vector $Q - P$. I would comment on the fact that one of the easiest ways to show a vector is in a plane is to dot it with the normal to the plane. (Brief comment only. No time to work out a problem.)

Problem 2 can be solved just by following the instructions. I don't know whether or not you need to draw the graphs of 2 planes to show they intersect in a line.

Problem 3 requires an introduction. I would draw $\vec{v}$, a plane, and the shadow of $\vec{v}$ on the plane. Then I would comment (using the graph) on subtracting the projection of $\vec{v}$ onto the normal of the plane from $\vec{v}$ itself to get the projection onto the plane.

As with most classes, I would draw my graphs before class begins. (While you are sketching, time is a'wasting!)