RECITATION 6

For the center of mass of an object at rest not to move, the sum of forces $\vec{F}$ on it must equal the zero vector. For an object at rest not to rotate, the sum of torques $\vec{\tau}$ about its center of mass must equal the zero vector.

1. Suppose a uniform thin rod lies along the y-axis from $y = -10$ to $y = 10$ and the following forces are applied to it: $\vec{F_1} =< 0, 0, -1 >$ applied at the point $(0, -5, 0)$, $\vec{F_2} =< 0, 0, 6 >$ applied at the point $(0, 5, 0)$, and $\vec{F_3} =< 0, 0, -5 >$ applied at the point $(0, y_0, 0)$ for some number $0 < y_0 < 10$.

(a) On x-y-z axes, sketch the beam and the forces on it.

(b) How can you tell the center of mass of the rod will not move?

(c) What would y_0 have to be in order for the rod not to rotate?

2. Forces give objects (linear) acceleration $\vec{a}$ according to Newton's law: $\vec{F} = m\vec{a}$
Torques give objects angular acceleration $\vec{\alpha}$ according to a rotational version of Newton's law:

$$\vec{\tau} = I\vec{\alpha} \qquad (*)$$

where I, the so-called moment of inertia, is a constant depending upon the distribution of mass of the object. To understand this, consider an object rotating about its center of mass with a certain rotational rate, say 10 revolutions per second. We say the magnitude of its **angular velocity** $\vec{\omega}$ is $||\vec{\omega}|| = 10*2\pi = 20\pi$ radians/sec (and the direction of $\vec{\omega}$ is in the direction of the axis of rotation). Then the **angular acceleration** $\vec{\alpha}$ is rate of change of angular velocity

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

Consequently, if an object initially at rest accelerates with a constant angular acceleration $\vec{\alpha}$, after t seconds it will be going with an angular velocity of

$$\vec{\omega} = \vec{\alpha}\, t \qquad (**)$$

We will assume in this problem that the moment of inertia I of the solid cylinder is 30 (in appropriate units). Equations (*) and (**) can be used to solve simple rotation problems.

(a) On x-y-z axes, sketch a solid ball, with its center of mass at the origin, of diameter 8 ft.

(b) Suppose force $\vec{F_1} =< 1, 0, 2 >$ (units in lbs.) is applied to the ball at point $(0, 4, 0)$ and force $\vec{F_2} =< -1, 0, -2 >$ is applied to the cylinder at point $(0, -4, 0)$. Place these forces in the sketch.

(c) Find each of the torques $\vec{\tau_1}$ and $\vec{\tau_2}$ generated by the forces, and then find their sum. (Recall $\vec{\tau} = \vec{r} \times \vec{F}$)

(d) Suppose the moment of inertia of the solid ball (in correct units) is 30. Find the angular acceleration $\vec{\alpha}$.

(e) Assuming the ball is initially at rest, find the number of revolutions per second it is making after 3 seconds. Also find a unit vector in the direction of the axis of rotation. Is the ball rotating about the z-axis? Why or why not?

3. Suppose a block weighing 10 lbs. is supported by two wires, one making an angle 30 degrees from (upward) vertical and the second making an angle of 45 degrees from the (upward) vertical. (The force of gravity and the two forces exerted by the wires will all be in the same plane)

(a) Assume the block remains stationary. Draw a diagram showing the block and all forces on it, giving labels to the **magnitudes** of the two forces exerted by the wires, and write simultaneous equations to find these magnitudes.

(b) We will not ask you to solve the simultaneous equations, as we are certain you could, especially if you were being paid to solve the problem – as some day you might be. However, if you did solve, and you took the sum of the magnitudes of the two forces being exerted on the block by the wires, it would add up to more than 10 lbs. How is this possible, if the block only weighs 10 lbs?

HINTS FOR INSTRUCTORS

Torques and rotation are difficult topics, even for engineering and physics majors. Even though the concepts involved, expecially in problem 2, are difficult, the actual calculations in problems 1 and 2 are quite simple.

Problem 2:

I have introduced angular velocity on the basis of something familiar to students, angular speed in revolutions per second. Obviously, one only need multiply by 2π to change revolutions per second into angular **speed**. Angular velocity, of course, is a vector, and I have given it as the vector pointing along the axis of rotation with magnitude equal to angular speed. I would prefer not to bring up the issue of "which" direction along the axis of rotation $\vec{\omega}$ points (ie., obeying right hand rule), unless one is backed into a corner by a student question. I have also chosen not to relate angular velocity to the derivative of θ, since, in the first place, it is not needed, and since this still leaves unclear the transition from the scalar derivative to the vector $\vec{\omega}$.

I would write the first and third displayed equations on the board. I would not write the second displayed equation, since it is not needed, and since, in any case, they have not yet studied the derivative of a vector function. (I have written it for the sake of comprehension, which, in my opinion, is helpful even if the formal concept, differentiation of a vector function, has not been introduced)

Problem 3:

I would roughly sketch the object and the wires in problem 3, with arrows going up the wires to indicate the tension forces. I probably would not draw a force vector for gravity, but I might remind them not to forget the force of gravity. Incidentally, for the answer to part (b), I personally would respond that the forces on the wires are, in part, pulling against each other, in addition to holding up the block.