RECITATION 10

1. Suppose an object weighing 8 lbs. is moving through a force field along the parabola $\vec{r}(t) =< t, \frac{1}{2}t^2, 0 >$ and when it is at the point $P = \vec{r}(1) =< 1, \frac{1}{2}, 0 >$ the force on it is known to be $\vec{F} =< 3, 8, 0 >$. Our engineering application demands we compute the speed of the object at the point P.

What makes this problem difficult is that the parameter t is not the time (Otherwise it would be easy to find the speed v by differentiation) but simply a convenient parametrization of the spiral path. Nevertheless, we can find the speed by using Newton's law in terms of our moving coordinate system. The trick is to treat t as the time in finding $\hat{T}$, $\hat{N}$, and κ, since these depend only on the geometry, and not the particular parametrization, so it will not matter that the expressions for $\vec{v}$ and $\vec{a}$ we compute along the way will actually be wrong (ie, they will be wrong but not $\hat{T}$, $\hat{N}$, κ).

(a) Find $\hat{T}(t)$ and $\hat{N}(t)$ and their values at $t = 1$. (Big hint: I computed $\hat{N}(t)$ at home, and got $\hat{N}(t) = \frac{1}{\sqrt{1+t^2}} < -t, 1, 0 >$. You can assume I am correct, but the cleverer guys among you – and there should be a lot of you, since we are always told the engineers are the smartest guys on campus – should check my result)

(b) Find (the wrong velocity) $\vec{v}(t)$ and $\vec{v}(1)$, and (the wrong acceleration) $\vec{a}(t)$ and $\vec{a}(1)$, and the norms $||\vec{v}(t)||$, $||\vec{v}(1)||$, and $||\vec{a}(1)||$, and use these (or any other method) to find the (correct) curvature κ.

(c) Now consider Newton's Law

$$\vec{F} = m\,\vec{a} = m[a_T \hat{T} + a_N \hat{N}] = m[(\frac{dv}{dt})\hat{T}(1) + \kappa\, v^2\, \hat{N}(1)].$$

Substitute on the left hand side and the right hand side for $\vec{F}$, κ, m, $\hat{T}$ and $\hat{N}$, and call the unknowns $\frac{dv}{dt} = \alpha$ and $v^2 = \beta$. Then, by setting the $\hat{\imath}$-components and the $\hat{\jmath}$-components equal on the left hand side and the right hand side of the equation, you should obtain two simultaneous equations for α and β. You need only solve for β. Remember that the mass of an 8 lb. weight is $m = w/g = \frac{8}{32} = \frac{1}{4}$ and not 8. The units of your answer v are determined by the choice of g, in this case feet/second. What, finally, do you get for the speed of the object at the point P?

2. Suppose a rocket is traveling in an elliptic orbit about the center of the earth (ie, not in a circular orbit) at a constant speed.

(a)) What can you say about a_T?

(b)) Can a_N be zero at a point of the orbit? Can it be constant throughout the orbit? How do you know?

(c)) Suppose you want to stay in the elliptic orbit, but want to arrange for a_N to be constant throughout the orbit. Can this be done, and, if so, how?

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