**Math 1224**

**Vector Geometry**

**RECITATION 8: Constant Acceleration**

**Goals:** Units 8 and 9 involve combining what we know about vectors with what we know about calculus to solve problems involving projectile motion. The problems in Unit 8 are somewhat simpler, because we'll consider only a single source of acceleration, gravity, which is a constant. In Unit 9, we'll cover similar problems in which the acceleration is variable. Some problems in this recitation ask you to set them up, but not solve them. This is to give your group the opportunity to set up several variations of a word problem in the time we have. You should be able to solve these problems fully at home.

**Introduction**

An object is thrown into the air with an initial velocity of $v_0$, at an angle of $\theta$. If we ignore the effects of air resistance, and assume there is no cross wind, then the object will move along a two-dimensional parabolic arc, as shown here. We can calculate the parametric equations for this arc by beginning with an acceleration vector that represents the downward force of gravity:

$$\vec{a}(t) = (0, -9.8) \text{ meters per second squared}$$

$$= (0, -32) \text{ feet per second squared}$$

$$= (0, -79000) \text{ miles per hour squared}$$

The correct units will be determined by the units given in the problem. Once we've determined the acceleration, we can determine the velocity by integrating:

$$\vec{v}(t) = \int_0^t \vec{a}(s) \, ds = (c_1, -9.8t + c_2) \text{ meters per second},$$

where $c_1$ and $c_2$ are the constants of integration. (The velocity will, of course, be different in other units.) We can solve for these constants because we know that the object had an initial velocity of $v_0$ and an initial angle of $\theta$. At time $t = 0$,

$$\vec{v}(0) = (c_1, -9.8(0) + c_2) = (v_0 \cos \theta, v_0 \sin \theta)$$

and so $c_1 = v_0 \cos \theta$ and $c_2 = v_0 \sin \theta$. We have to integrate a second time to find the position ($\vec{r}$) of the object:

$$\vec{r}(t) = \int_0^t \vec{v}(s) \, ds = \left( (v_0 \cos \theta)t + d_1, -4.9t^2 + (v_0 \sin \theta)t + d_2 \right) \text{ meters}.$$ 

Be careful here—$\theta$ is a constant, not a variable, so the antiderivative of $\cos \theta$ is $(\cos \theta)t$, not $\sin \theta$!

We'll need to know the starting position of the object in order to solve for $d_1$ and $d_2$; we usually let the starting position be $(0, h)$, where $h$ is the initial height of the object above the ground.
Work with your group members to solve these problems. Completely solve problems 1 and 5, but you need only set up problems 2–4.

**Problem 1: Tranquilizing the elephant**

In order to safely treat a wounded elephant lying 100 feet away at ground level, a doctor must first tranquilize the elephant. The doctor has a tranquilizer gun mounted to her vehicle at a fixed angle of elevation of 60°. If we assume the doctor is firing from ground level when she fires the gun, what muzzle speed is required to tranquilize the elephant?

**Problem 2: Tranquilizing the elephant on the hill**

In this problem, suppose that the wounded elephant is lying 50 feet away at the top of a 10-foot hill. How does this change the problem? Write an equation that will determine the correct muzzle speed. You do not have to solve this equation.

**Problem 3: Tranquilizing the angry elephant**

In this problem, suppose that the wounded elephant is situated as in Problem 1 (100 feet away, at ground level). But this time, the elephant is not incapacitated, and instead of lying on the ground, it is moving towards you at a speed of 5 feet per second. How does this change the problem? Write an equation that will determine the correct muzzle speed. You do not have to solve this equation.

**Problem 4: Tranquilizing the elephant with a different gun**

In this problem, suppose that the wounded elephant is situated as in Problem 1 (100 feet away, at ground level). But this time, suppose the doctor’s tranquilizer gun was not fixed at a 60° angle, but instead the gun had a fixed muzzle speed of 80 feet per second. Write an equation that will determine the correct firing angle. You do not have to solve this equation. (If you wish to solve this equation, the equation has two solutions: Ordinarily a doctor in this situation would choose the smaller firing angle, since it results in a faster, more direct trajectory.)

**Problem 5: An elephant too far away**

If in problem 4, the elephant were 1000 feet away instead of 100 feet away, you would not have been able to solve the problem. This is because a tranquilizer gun with a muzzle speed of 80 feet per second has a maximum range that is less than 1000 feet. Find the maximum range of the gun.

*Hint 1: Find a formula for the range of the gun. Hint 2: To find a maximum for an expression, we look for values that make the derivative equal to zero. In this problem, the firing angle θ is no longer a constant—it’s now a variable, since different angles will affect the range. We want to find the angle that gives the greatest range.*